

**Directions** Answer all questions in the space provided and box your final answers. Good luck!

1. (8 points) Let  $f$  be defined to be the one-to-one function

$$f(x) = \sqrt{x^3 + x^2 + x + 1}.$$

Use the inverse function theorem to find  $(f^{-1})'(2)$ .

*Hint: You can easily find  $f^{-1}(2)$  by just guessing and checking.*

2. (8 points) Suppose a sample of radioactive material decays exponentially, and suppose it takes 6 days for the sample to decay to 88% of its initial mass. Find the half-life of the material.

3. Differentiate each of the following functions.

(a) (8 points)  $f(x) = \tan^{-1}(\sinh x)$

(b) (8 points)  $g(x) = \frac{2^x}{x^2\sqrt{x^4+1}}$  *Hint: Use logarithmic differentiation.*

4. (8 points) Calculate the following limit.

$$\lim_{x \rightarrow 0^+} \ln x \sin x$$

5. Evaluate each of the following integrals.

(a) (8 points)  $\int_1^4 \sqrt{t} \ln t \, dt$

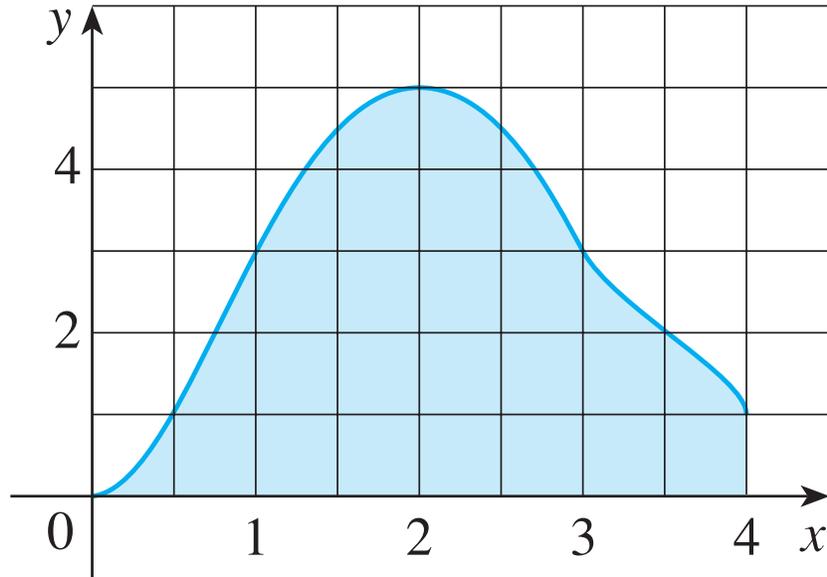
(b) (8 points)  $\int \sin^5 x \cos^3 x \, dx$

6. Evaluate each of the following integrals.

(a) (8 points)  $\int \frac{1}{x^2\sqrt{x^2+9}} dx$

(b) (8 points)  $\int \frac{10}{(x-1)(x^2+9)} dx$

7. Approximate the area under the graph by using



(a) (4 points) the trapezoid rule with  $n = 4$  (i.e.  $T_4$ ), and

(b) (4 points) Simpson's rule with  $n = 4$  (i.e.  $S_4$ ).

8. (8 points) Evaluate the following improper integral.

$$\int_1^{\infty} \frac{\ln x}{x^3} dx$$

*Hint: First use integration by parts to evaluate the indefinite integral.*