Name: *ANSWER KEY* 3/5/2015

Math 202 Quiz 1

Directions Answer all questions in the space provided and box your final answers. Good luck!

1. (4 points) Which of the following is correct?

$$\underbrace{\cos(\cos^{-1}(\pi/2))}_{\text{UNDEFINED}} = \pi/2 \qquad \sin^{-1}(\sin(3\pi/2)) \neq 3\pi/2 = -\frac{\pi}{2} \qquad \cos^{-1}(\cos(\pi/2)) = \pi/2$$

2. (4 points) Find $(f^{-1})'(a)$, where $f(x) = 4x^5 + 2x - 1$ and $a = 5$.

SINCE $f(1) = 5$, THE INVERSE FUNCTION THM GIVES

$$(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))} = \frac{1}{f'(1)} = \boxed{\frac{1}{22}}$$

$$\left(f'(x) = 20x^4 + 2 \right)$$

3. (4 points) Suppose a sample of radioactive material takes 6 days to decay to 85% of its original mass. Assuming exponential decay, find the *half-life* of this material.

$$M(t) = M_0 (.85)^{t/6} = \frac{1}{2} M_0$$

$$\frac{t}{6} \ln(.85) = \ln\left(\frac{1}{2}\right)$$

$$t = \boxed{\frac{6 \ln(\frac{1}{2})}{\ln(.85)}}$$

4. Differentiate the following functions.

(a) (4 points) $f(x) = \ln\left(\frac{e^x}{x}\right)$ $= \ln(e^x) - \ln(x) = x - \ln(x)$

$$\boxed{f'(x) = 1 - \frac{1}{x}}$$

(b) (4 points) $g(x) = \pi^{x^2}$

$$\boxed{g'(x) = \pi^{x^2} \ln(\pi) \cdot 2x}$$

JUST RECALL: $\frac{d}{dx}[a^x] = a^x \ln a$

+ CHAIN RULE:

$$\frac{d}{dx}[a^{f(x)}] = a^{f(x)} \ln a \cdot f'(x)$$

5. Evaluate the following integral.

$$\int_0^{\pi/2} \frac{\cos x}{4 + \sin^2 x}\, dx$$

LET $u = \sin x$

$du = \cos x \, dx$

$$\rightsquigarrow \int_0^1 \frac{1}{4 + u^2}\, du = \frac{1}{2} \tan^{-1}\left(\frac{u}{2}\right)\Big|_0^1$$

$$= \frac{1}{2}\left(\tan^{-1}\left(\frac{1}{2}\right) - \underbrace{\tan^{-1}(0)}_{0}\right)$$

$$= \boxed{\frac{1}{2} \tan^{-1}\left(\frac{1}{2}\right)}$$

6. Evaluate each of the following limits.

(a) (4 points) $\lim_{x\to\infty} \frac{4x^2+3x-e^x}{3x^2+e^x}$ $:\ \frac{\infty}{\infty}$ IND. FORM

$$\xrightarrow{L'H\hat{O}} \lim_{x\to\infty} \frac{8x+3-e^x}{6x+e^x} \ :\ \frac{\infty}{\infty} \text{ IND. FORM}$$

$$\xrightarrow{L'H\hat{O}} \lim_{x\to\infty} \frac{8-e^x}{6+e^x} \ :\ \frac{\infty}{\infty} \text{ IND. FORM}$$

$$\xrightarrow{L'H\hat{O}} \lim_{x\to\infty} \frac{-e^x}{e^x} = \boxed{-1}$$

(b) (4 points) $\lim_{t\to\infty} \left(1+\frac{1}{t}\right)^t$ $:\ 1^{\infty}$ IND. FORM

$$= \lim_{t\to\infty} \mathrm{EXP}\left[\ln\left(\left(1+\frac{1}{t}\right)^t \right) \right]$$

$\frac{0}{0}$ IND. FORM

$$= \mathrm{EXP}\left[\lim_{t\to\infty} t \ln\left(1+\frac{1}{t}\right) \right] = \mathrm{EXP}\left[\lim_{t\to\infty} \frac{\ln\left(1+\frac{1}{t}\right)}{\frac{1}{t}} \right]$$

$$\xrightarrow{L'H\hat{O}} \mathrm{EXP}\left[\lim_{t\to\infty} \frac{\frac{1}{1+\frac{1}{t}}\left(\frac{-1}{t^2}\right)}{-\frac{1}{t^2}} \right]$$

$$= \mathrm{EXP}\left[\lim_{t\to\infty} \frac{1}{1+\frac{1}{t}} \right] = \mathrm{EXP}(1) = e^1 = \boxed{e}$$