Name: *ANSWER KEY* 3/24/2015

Math 202 Quiz 2

Directions Answer all questions in the space provided and box your final answers. Good luck!

1. Evaluate the following integrals.

   (a) (8 points) $\int x^5 \ln x \, dx$

LET $u = \ln x$ $\quad v = \frac{1}{6}x^6$

$du = \frac{1}{x}dx$ $\quad dv = x^5 dx$

$$= \frac{1}{6}x^6 \ln x - \frac{1}{6}\int x^5 dx = \boxed{\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C}$$

OR $\frac{1}{6}x^6\left(\ln x - \frac{1}{6}\right) + C$

   (b) (8 points) $\int_0^\pi e^x \sin x \, dx$

LET $u = e^x$ $\quad v = -\cos x$

$du = e^x dx$ $\quad dv = \sin x \, dx$

$$= -e^x \cos x \Big|_0^\pi + \int_0^\pi e^x \cos x \, dx$$

LET $u = e^x$ $\quad v = \sin x$

$du = e^x dx$ $\quad dv = \cos x \, dx$

ZERO

$$\int_0^\pi e^x \sin x \, dx = -e^x \cos x \Big|_0^\pi + e^x \sin x \Big|_0^\pi - \int_0^\pi e^x \sin x \, dx$$

$$2\int_0^\pi e^x \sin x \, dx = -e^x \cos x \Big|_0^\pi$$

$$\int_0^\pi e^x \sin x \, dx = -\frac{1}{2}e^x \cos x \Big|_0^\pi = -\frac{1}{2}\left(-e^\pi - 1\right) = \boxed{\frac{e^\pi + 1}{2}}$$

2. Evaluate the following integrals.

(a) (8 points) $\int \sin^5(\pi\theta)\, d\theta$

$$= \int \left(\sin^2(\pi\theta)\right)^2 \sin(\pi\theta)\, d\theta = \int \left(1-\cos^2(\pi\theta)\right)^2 \sin(\pi\theta)\, d\theta$$

Let $u = \cos(\pi\theta)$

$-\frac{1}{\pi}\, du = \sin(\pi\theta)\, d\theta$

$$\rightarrow -\frac{1}{\pi}\int (1-u^2)^2\, du$$

$$= -\frac{1}{\pi}\int 1 - 2u^2 + u^4\, du$$

$$= -\frac{1}{\pi}\left(u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right) + C$$

$$\rightarrow \boxed{-\frac{1}{\pi}\left(\cos(\pi\theta) - \frac{2}{3}\cos^3(\pi\theta) + \frac{1}{5}\cos^5(\pi\theta)\right) + C}$$

(b) (8 points) $\int \tan^3(2\theta)\sec^5(2\theta)\, d\theta$

$$= \int \tan^2(2\theta)\sec^4(2\theta)\sec(2\theta)\tan(2\theta)\, d\theta$$

$$= \int (\sec^2(2\theta) - 1)\sec^4(2\theta)\sec(2\theta)\tan(2\theta)\, d\theta$$

Let $u = \sec(2\theta)$

$\frac{1}{2}\, du = \sec(2\theta)\tan(2\theta)\, d\theta$

$$\rightarrow \frac{1}{2}\int (u^2-1)u^4\, du = \frac{1}{2}\int u^6 - u^4\, du = \frac{1}{14}u^7 - \frac{1}{10}u^5 + C$$

$$\rightarrow \boxed{\frac{1}{14}\sec^7(2\theta) - \frac{1}{10}\sec^5(2\theta) + C}$$

3. (8 points) Use trigonometric substitution to evaluate the integral

$$\int \frac{\sqrt{x^2 - a^2}}{x^4}\, dx.$$

LET $x = a \sec\theta$ $\rightsquigarrow \displaystyle\int \frac{a \tan\theta}{a^4 \sec^4\theta}\, a \sec\theta \tan\theta \, d\theta$

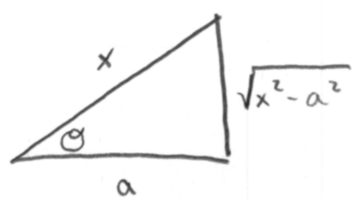
$dx = a \sec\theta \tan\theta \, d\theta$

$$= \frac{1}{a^2} \int \frac{\tan^2\theta}{\sec^3\theta}\, d\theta = \frac{1}{a^2} \int \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^3\theta \, d\theta$$

$$= \frac{1}{a^2} \int \underbrace{\sin^2\theta}_{u^2} \, \underbrace{\cos\theta \, d\theta}_{du} = \frac{1}{3a^2} \sin^3\theta + C$$

RECALL: $x = a \sec\theta$

$\frac{x}{a} = \sec\theta \longrightarrow \cos\theta = \frac{a}{x}$



$\therefore \sin\theta = \frac{\sqrt{x^2-a^2}}{x}$

$$\rightsquigarrow \boxed{\frac{1}{3a^2}\left(\frac{\sqrt{x^2-a^2}}{x}\right)^3 + C}$$

4. (8 points) Use the method of partial fractions to evaluate the integral

$$\int \frac{1}{x^4 - x^2}\,dx.$$

$$x^4 - x^2 = x^2(x^2-1) = x^2(x+1)(x-1)$$

WRITE $\dfrac{1}{x^2(x+1)(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} + \dfrac{D}{x-1}$

$$1 = Ax(x+1)(x-1) + B(x+1)(x-1) + Cx^2(x-1) + Dx^2(x+1)$$

$x=0$: $\quad 1 = -B \quad$ so $\quad \boxed{B = -1}$

$x=-1$: $\quad 1 = -2C \quad$ so $\quad \boxed{C = -\frac{1}{2}}$

$x=1$: $\quad 1 = 2D \quad$ so $\quad \boxed{D = \frac{1}{2}}$

NOW WE HAVE $\quad 1 = Ax(x+1)(x-1) - (x+1)(x-1) - \frac{1}{2}x^2(x-1) + \frac{1}{2}x^2(x+1)$

$x=3$: $\quad 1 = 24A - 8 - 9 + 18 = 24A + 1$

$0 = 24A \quad$ so $\quad \boxed{A = 0}$

$$\therefore \int \frac{1}{x^4-x^2}\,dx = \int \frac{-1}{x^2} - \frac{1/2}{x+1} + \frac{1/2}{x-1}\,dx$$

$$= \boxed{\frac{1}{x} - \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C \quad \text{or} \quad \frac{1}{x} + \ln\sqrt{\left|\frac{x-1}{x+1}\right|} + C}$$