Name: *ANSWER KEY* 4/30/2015

Math 202 Quiz 3

Directions Answer all questions in the space provided and box your final answers. Good luck!

1. Use the comparison theorem to determine if each of the following integrals converges or diverges.

(a) (8 points) $\int_1^\infty \frac{1+\sin^2 x}{\sqrt{x}}\,dx$

$$\int_1^\infty \frac{1+\sin^2 x}{\sqrt{x}}\,dx \geq \int_1^\infty \frac{1}{\sqrt{x}}\,dx = \infty$$

"DIVERGES" BY p-TEST WITH $p = \frac{1}{2}$.

$\therefore$ THE INTEGRAL DIVERGES BY COMPARISON THM.

(b) (8 points) $\int_1^\infty \frac{\arctan x}{\sqrt{1+x^3}}\,dx$

$$\int_1^\infty \frac{\arctan x}{\sqrt{1+x^3}}\,dx \leq \int_1^\infty \frac{\pi/2}{\sqrt{1+x^3}}\,dx \leq \frac{\pi}{2}\int_1^\infty \frac{1}{\sqrt{x^3}}\,dx < \infty$$

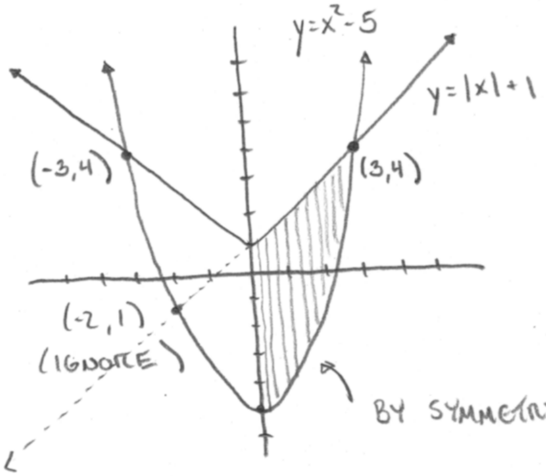
"CONVERGES" BY p-TEST WITH $p = \frac{3}{2}$.

$\therefore$ THE INTEGRAL CONVERGES BY COMPARISON THM.

2. (8 points) Sketch the region enclosed by the curves

$$y = |x| + 1 \quad \text{and} \quad y = x^2 - 5,$$

and find the area of the region.



INTERSECTION: $x+1 = x^2-5$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

BY SYMMETRY, $A = 2\int_0^3 (x+1) - (x^2-5)\,dx$

$$A = 2\int_0^3 x - x^2 + 6\,dx = 2\left[\frac{1}{2}x^2 - \frac{1}{3}x^3 + 6x\right]_0^3$$

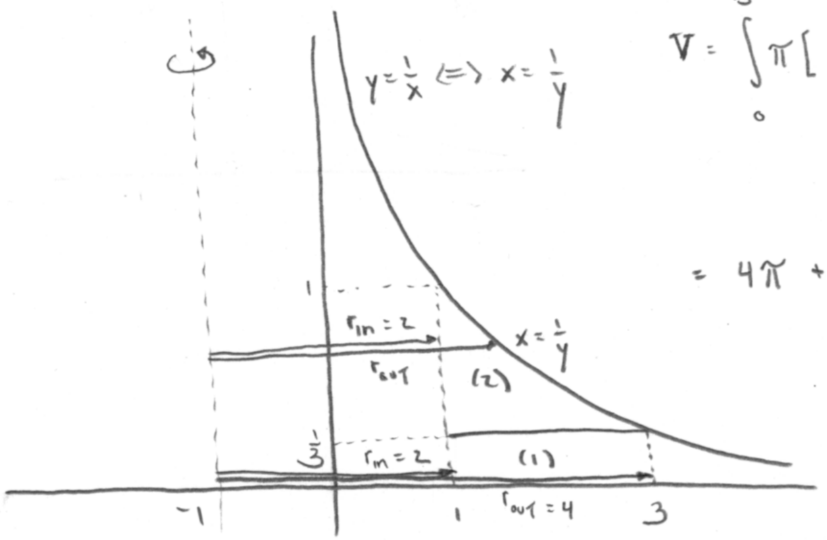
$$= 2\left[\frac{9}{2} - 9 + 18\right] = \boxed{27}$$

3. Sketch the region enclosed by the curves

$$y = \frac{1}{x}, \quad y = 0, \quad x = 1, \quad \text{and } x = 3$$

and find the volume of the solid obtained by rotating the region about the the line $x = -1$ in two ways.

(a) (8 points) By using the method of disks and washers.



$$V = \int_0^{1/3} \pi \left[4^2 - 2^2 \right] dy + \int_{1/3}^{1} \pi \left[\left(1 + \frac{1}{y}\right)^2 - 2^2 \right] dy$$

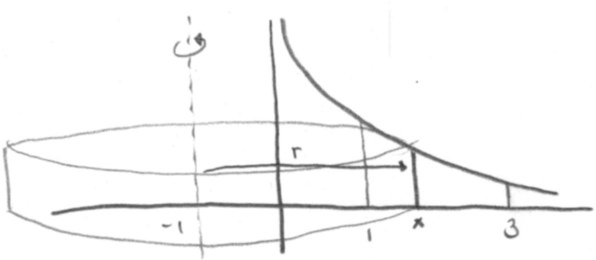
(1) (2)

$$= 4\pi + \pi \int_{1/3}^{1} \frac{2}{y} + \frac{1}{y^2} - 3 \, dy$$

$$= 4\pi + \pi \left[2 \operatorname{Log} y - \frac{1}{y} - 3y \right]_{1/3}^{1}$$

$$= 4\pi + \pi \left[\underbrace{2 \operatorname{Log} 1}_{0} - 1 - 3 - \left(\underbrace{2 \operatorname{Log} \tfrac{1}{3}}_{-2 \operatorname{Log} 3} - 3 - 1 \right) \right] = \pi \left(4 + 2 \operatorname{Log} 3 \right)$$

(b) (8 points) By using the method of cylindrical shells.



$$V = \int_1^3 \underbrace{2\pi (x+1)}_{\text{CIRCUMF.}} \underbrace{\frac{1}{x}}_{\text{HEIGH}} \underbrace{dx}_{\text{THICKNESS}}$$

$$= 2\pi \int_1^3 1 + \frac{1}{x} \, dx = 2\pi \left[x + \operatorname{Log} x \right]_1^3$$

$$= 2\pi \left[3 + \operatorname{Log} 3 - 1 - \underbrace{\operatorname{Log} 1}_{0} \right] = 2\pi \left(2 + \operatorname{Log} 3 \right)$$

(SAME!)

4. (8 points) Find the length of the curve

$$y = \frac{x^3}{4} + \frac{1}{3x}, \quad 1 \le x \le 4.$$

$$y' = \frac{3}{4}x^2 - \frac{1}{3x^2} \longrightarrow (y')^2 = \frac{9}{16}x^4 \ominus \frac{1}{2} + \frac{1}{9x^4}$$

$$L = \int_1^4 \sqrt{1 + (y')^2}\, dx = \int_1^4 \sqrt{\frac{9}{16}x^4 \oplus \frac{1}{2} + \frac{1}{9x^4}}\, dx$$

$$= \int_1^4 \sqrt{\left(\frac{3}{4}x^2 + \frac{1}{3x^2}\right)^2}\, dx$$

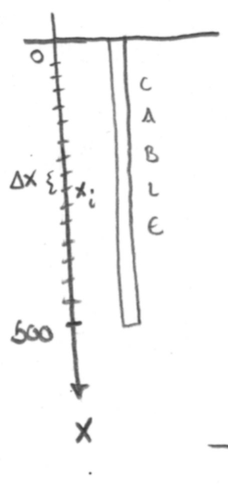
$$= \int_1^4 \frac{3}{4}x^2 + \frac{1}{3x^2}\, dx = \frac{1}{4}x^3 - \frac{1}{3x}\Bigg|_1^4$$

$$= \left(16 - \frac{1}{12}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)$$

$$= 16 - \frac{1}{12} + \frac{1}{12} = \boxed{16}$$

5. (8 POINTS) A CABLE THAT WEIGHS 2 LB/FT IS USED TO LIFT 800 LB OF COAL UP A MINE SHAFT 500 FT DEEP. FIND THE WORK DONE.

WORK DONE LIFTING CABLE



$$W_i = Fd = \underbrace{(2\ \text{LB/ft})(\Delta x\ \text{ft})}_{\text{WEIGHT OF } i^{TH} \text{ PIECE OF CABLE}} \overbrace{(x_i)}^{\text{DISTANCE TO TOP OF MINE}}$$

$$\rightarrow W = \int_0^{500} 2x\,dx = x^2 \Big|_0^{500} = 250{,}000 \text{ FT-LBS.}$$

WORK DONE LIFTING COAL

$$W = Fd = (800\ \text{LBS})(500\ \text{ft}) = 400{,}000 \text{ FT-LBS}$$

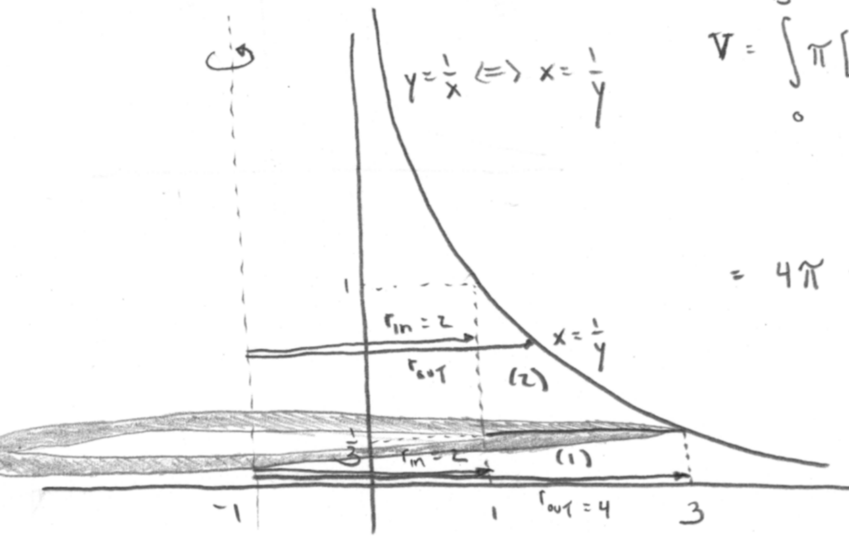
$\therefore$ TOTAL WORK = 250000 + 400000 = $\boxed{650{,}000 \text{ FT-LBS}}$

3. Sketch the region enclosed by the curves

$$y = \frac{1}{x}, \quad y = 0, \quad x = 1, \quad \text{and} \quad x = 3$$

and find the volume of the solid obtained by rotating the region about the the line $x = -1$ in two ways.

(a) (8 points) By using the method of disks and washers.



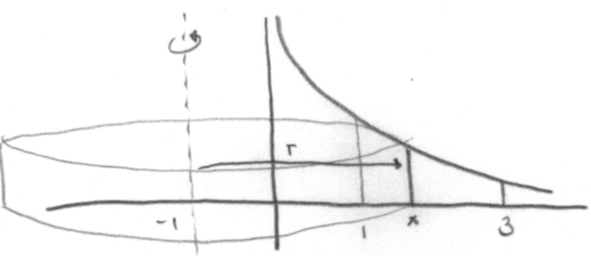
$$V = \underbrace{\int_0^{1/3} \pi\left[4^2 - 2^2\right] dy}_{(1)} + \underbrace{\int_{1/3}^{1} \pi\left[\left(1+\frac{1}{y}\right)^2 - 2^2\right] dy}_{(2)}$$

$$= 4\pi + \pi \int_{1/3}^{1} \frac{2}{y} + \frac{1}{y^2} - 3 \, dy$$

$$= 4\pi + \pi\left[2\operatorname{Log} y - \frac{1}{y} - 3y\right]_{1/3}^{1}$$

$$= 4\pi + \pi\Big[\underbrace{2\operatorname{Log} 1}_{0} - 1 - 3 - \big(\underbrace{2\operatorname{Log}\tfrac{1}{3}}_{-2\operatorname{Log} 3} - 3 - 1\big)\Big] = \pi\left(4 + 2\operatorname{Log} 3\right)$$

(b) (8 points) By using the method of cylindrical shells.



$$V = \int_1^3 \underbrace{2\pi(x+1)}_{\text{CIRCUMF.}} \underbrace{\frac{1}{x}}_{\text{HEIGHT}} \underbrace{dx}_{\text{THICKNESS}}$$

$$= 2\pi \int_1^3 1 + \frac{1}{x} \, dx = 2\pi\left[x + \operatorname{Log} x\right]_1^3$$

$$= 2\pi\Big[3 + \operatorname{Log} 3 - 1 - \underbrace{\operatorname{Log} 1}_{0}\Big] = 2\pi\left(2 + \operatorname{Log} 3\right)$$

(SAME!)