which has the solution $A = 1$, $B = -1$, $C = -1$, $D = 1$, and $E = 0$. Thus

$$\int \frac{1 - x + 2x^2 - x^3}{x(x^2 + 1)^2}\,dx = \int \left(\frac{1}{x} - \frac{x + 1}{x^2 + 1} + \frac{x}{(x^2 + 1)^2}\right) dx$$

$$= \int \frac{dx}{x} - \int \frac{x}{x^2 + 1}\,dx - \int \frac{dx}{x^2 + 1} + \int \frac{x\,dx}{(x^2 + 1)^2}$$

▪ In the second and fourth terms we made the mental substitution $u = x^2 + 1$.

$$= \ln|x| - \tfrac{1}{2}\ln(x^2 + 1) - \tan^{-1}x - \frac{1}{2(x^2 + 1)} + K$$

■

6.3 EXERCISES

1–6 ▪ Write out the form of the partial fraction decomposition of the function (as in Example 6). Do not determine the numerical values of the coefficients.

1. (a) $\dfrac{2x}{(x + 3)(3x + 1)}$ (b) $\dfrac{1}{x^3 + 2x^2 + x}$

2. (a) $\dfrac{x - 1}{x^3 + x^2}$ (b) $\dfrac{x - 1}{x^3 + x}$

3. (a) $\dfrac{2}{x^2 + 3x - 4}$ (b) $\dfrac{x^2}{(x - 1)(x^2 + x + 1)}$

4. (a) $\dfrac{x^3}{x^2 + 4x + 3}$ (b) $\dfrac{2x + 1}{(x + 1)^3(x^2 + 4)^2}$

5. (a) $\dfrac{x^4}{x^4 - 1}$ (b) $\dfrac{t^4 + t^2 + 1}{(t^2 + 1)(t^2 + 4)^2}$

6. (a) $\dfrac{x^4}{(x^3 + x)(x^2 - x + 3)}$ (b) $\dfrac{1}{x^6 - x^3}$

7–34 ▪ Evaluate the integral.

7. $\displaystyle\int \frac{x}{x - 6}\,dx$

8. $\displaystyle\int \frac{r^2}{r + 4}\,dr$

9. $\displaystyle\int \frac{x - 9}{(x + 5)(x - 2)}\,dx$

10. $\displaystyle\int \frac{1}{(t + 4)(t - 1)}\,dt$

11. $\displaystyle\int_2^3 \frac{1}{x^2 - 1}\,dx$

12. $\displaystyle\int_0^1 \frac{x - 1}{x^2 + 3x + 2}\,dx$

13. $\displaystyle\int \frac{ax}{x^2 - bx}\,dx$

14. $\displaystyle\int \frac{1}{(x + a)(x + b)}\,dx$

15. $\displaystyle\int_0^1 \frac{2x + 3}{(x + 1)^2}\,dx$

16. $\displaystyle\int_0^1 \frac{x^3 - 4x - 10}{x^2 - x - 6}\,dx$

17. $\displaystyle\int_1^2 \frac{4y^2 - 7y - 12}{y(y + 2)(y - 3)}\,dy$

18. $\displaystyle\int \frac{x^2 + 2x - 1}{x^3 - x}\,dx$

19. $\displaystyle\int \frac{1}{(x + 5)^2(x - 1)}\,dx$

20. $\displaystyle\int \frac{x^2}{(x - 3)(x + 2)^2}\,dx$

21. $\displaystyle\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2}\,dx$

22. $\displaystyle\int \frac{x^2 - x + 6}{x^3 + 3x}\,dx$

23. $\displaystyle\int \frac{10}{(x - 1)(x^2 + 9)}\,dx$

24. $\displaystyle\int \frac{x^2 - 2x - 1}{(x - 1)^2(x^2 + 1)}\,dx$

25. $\displaystyle\int \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)(x^2 + 2)}\,dx$

26. $\displaystyle\int \frac{x^3 - 2x^2 + x + 1}{x^4 + 5x^2 + 4}\,dx$

27. $\displaystyle\int \frac{x + 4}{x^2 + 2x + 5}\,dx$

28. $\displaystyle\int_0^1 \frac{x}{x^2 + 4x + 13}\,dx$

29. $\displaystyle\int \frac{1}{x^3 - 1}\,dx$

30. $\displaystyle\int \frac{x^3}{x^3 + 1}\,dx$

31. $\displaystyle\int \frac{dx}{x^4 - x^2}$

32. $\displaystyle\int_0^1 \frac{2x^3 + 5x}{x^4 + 5x^2 + 4}\,dx$

33. $\displaystyle\int \frac{x - 3}{(x^2 + 2x + 4)^2}\,dx$

34. $\displaystyle\int \frac{x^4 + 1}{x(x^2 + 1)^2}\,dx$

35–40 ▪ Make a substitution to express the integrand as a rational function and then evaluate the integral.

35. $\displaystyle\int_9^{16} \frac{\sqrt{x}}{x - 4}\,dx$ (Let $u = \sqrt{x}$.)

36. $\displaystyle\int_0^1 \frac{1}{1 + \sqrt[3]{x}}\,dx$ (Let $u = \sqrt[3]{x}$.)

37. $\displaystyle\int \frac{x^3}{\sqrt[3]{x^2 + 1}}\,dx$

38. $\displaystyle\int_{1/3}^3 \frac{\sqrt{x}}{x^2 + x}\,dx$

39. $\displaystyle\int \frac{e^{2x}}{e^{2x} + 3e^x + 2}\,dx$

40. $\displaystyle\int \frac{\cos x}{\sin^2 x + \sin x}\,dx$

41–42 ▪ Use integration by parts, together with the techniques of this section, to evaluate the integral.

41. $\int \ln(x^2 - x + 2)\, dx$

42. $\int x \tan^{-1}x\, dx$

43. One method of slowing the growth of an insect population without using pesticides is to introduce into the population a number of sterile males that mate with fertile females but produce no offspring. If P represents the number of female insects in a population, S the number of sterile males introduced each generation, and r the population's natural growth rate, then the female population is related to time t by

$$t = \int \frac{P + S}{P[(r - 1)P - S]}\, dP$$

Suppose an insect population with 10,000 females grows at a rate of $r = 0.10$ and 900 sterile males are added. Evaluate the integral to give an equation relating the female population to time. (Note that the resulting equation can't be solved explicitly for P.)

44. Factor $x^4 + 1$ as a difference of squares by first adding and subtracting the same quantity. Use this factorization to evaluate $\int 1/(x^4 + 1)\, dx$.

45. Suppose that F, G, and Q are polynomials and

$$\frac{F(x)}{Q(x)} = \frac{G(x)}{Q(x)}$$

for all x except when $Q(x) = 0$. Prove that $F(x) = G(x)$ for all x. [*Hint:* Use continuity.]

46. If f is a quadratic function such that $f(0) = 1$ and

$$\int \frac{f(x)}{x^2(x + 1)^3}\, dx$$

is a rational function, find the value of $f'(0)$.

6.4 INTEGRATION WITH TABLES AND COMPUTER ALGEBRA SYSTEMS

In this section we describe how to evaluate integrals using tables and computer algebra systems.

TABLES OF INTEGRALS

Tables of indefinite integrals are very useful when we are confronted by an integral that is difficult to evaluate by hand and we don't have access to a computer algebra system. A relatively brief table of 120 integrals, categorized by form, is provided on the Reference Pages at the back of the book. More extensive tables are available in *CRC Standard Mathematical Tables and Formulae,* 31st ed, by Daniel Zwillinger (Boca Raton, FL: CRC Press, 2002) (709 entries) or in Gradshteyn and Ryzhik's *Table of Integrals, Series, and Products,* 6e, edited by A. Jefferey and D. Zwillinger (San Diego: Academic Press, 2000), which contains hundreds of pages of integrals. It should be remembered, however, that integrals do not often occur in exactly the form listed in a table. Usually we need to use the Substitution Rule or algebraic manipulation to transform a given integral into one of the forms in the table.

▪ The Table of Integrals appears on Reference Pages 6–10 at the back of the book.

V EXAMPLE 1 Use the Table of Integrals to find $\displaystyle\int \frac{x^2}{\sqrt{5 - 4x^2}}\, dx$.

SOLUTION If we look at the section of the table entitled *Forms involving* $\sqrt{a^2 - u^2}$, we see that the closest entry is number 34:

$$\int \frac{u^2}{\sqrt{a^2 - u^2}}\, du = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$$