Formula 5 gives

$$L = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{2\pi} \sqrt{(1 + \sin\theta)^2 + \cos^2\theta}\; d\theta$$

$$= \int_0^{2\pi} \sqrt{2 + 2\sin\theta}\; d\theta$$

We could evaluate this integral by multiplying and dividing the integrand by $\sqrt{2 - 2\sin\theta}$, or we could use a computer algebra system. In any event, we find that the length of the cardioid is $L = 8$. ■

9.4 EXERCISES

1–4 ▪ Find the area of the region that is bounded by the given curve and lies in the specified sector.

1. $r = \sqrt{\theta}$, $0 \leqslant \theta \leqslant \pi/4$

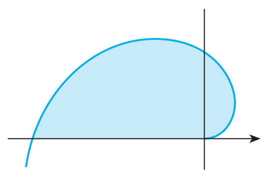
2. $r = e^{\theta/2}$, $\pi \leqslant \theta \leqslant 2\pi$

3. $r = \sin\theta$, $\pi/3 \leqslant \theta \leqslant 2\pi/3$

4. $r = \sqrt{\sin\theta}$, $0 \leqslant \theta \leqslant \pi$

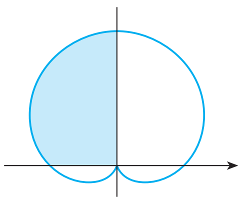
5–8 ▪ Find the area of the shaded region.

5.



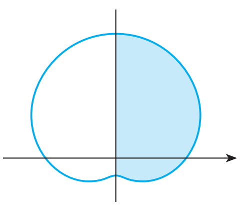
$r = \theta$

6.



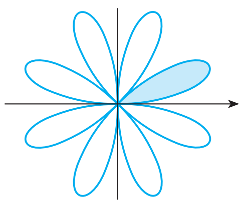
$r = 1 + \sin\theta$

7.



$r = 4 + 3\sin\theta$

8.



$r = \sin 4\theta$

9–12 ▪ Sketch the curve and find the area that it encloses.

9. $r^2 = 4\cos 2\theta$

10. $r = 3(1 + \cos\theta)$

11. $r = 2\cos 3\theta$

12. $r = 2 + \cos 2\theta$

13–14 ▪ Graph the curve and find the area that it encloses.

13. $r = 1 + 2\sin 6\theta$

14. $r = 2\sin\theta + 3\sin 9\theta$

15–18 ▪ Find the area of the region enclosed by one loop of the curve.

15. $r = \sin 2\theta$

16. $r = 4\sin 3\theta$

17. $r = 1 + 2\sin\theta$ (inner loop)

18. $r = 2\cos\theta - \sec\theta$

19–22 ▪ Find the area of the region that lies inside the first curve and outside the second curve.

19. $r = 4\sin\theta$, $r = 2$

20. $r = 1 - \sin\theta$, $r = 1$

21. $r = 3\cos\theta$, $r = 1 + \cos\theta$

22. $r = 2 + \sin\theta$, $r = 3\sin\theta$

23–26 ▪ Find the area of the region that lies inside both curves.

23. $r = \sin\theta$, $r = \cos\theta$

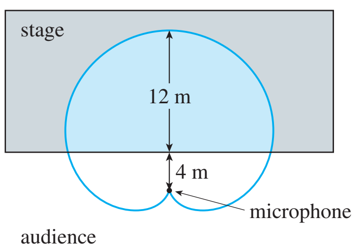
24. $r = \sin 2\theta$, $r = \sin\theta$

25. $r = \sin 2\theta$, $r = \cos 2\theta$

26. $r^2 = 2\sin 2\theta$, $r = 1$

27. Find the area inside the larger loop and outside the smaller loop of the limaçon $r = \frac{1}{2} + \cos\theta$.

28. When recording live performances, sound engineers often use a microphone with a cardioid pickup pattern because it suppresses noise from the audience. Suppose the microphone is placed 4 m from the front of the stage (as in the figure) and the boundary of the optimal pickup region is



given by the cardioid $r = 8 + 8 \sin \theta$, where r is measured in meters and the microphone is at the pole. The musicians want to know the area they will have on stage within the optimal pickup range of the microphone. Answer their question.

29–32 ▪ Find all points of intersection of the given curves.

29. $r = \cos \theta, \quad r = 1 - \cos \theta$

30. $r = \cos 3\theta, \quad r = \sin 3\theta$

31. $r = \sin \theta, \quad r = \sin 2\theta$

32. $r^2 = \sin 2\theta, \quad r^2 = \cos 2\theta$

33–36 ▪ Find the exact length of the polar curve.

33. $r = 3 \sin \theta, \quad 0 \leqslant \theta \leqslant \pi/3$

34. $r = e^{2\theta}, \quad 0 \leqslant \theta \leqslant 2\pi$

35. $r = \theta^2, \quad 0 \leqslant \theta \leqslant 2\pi$

36. $r = \theta, \quad 0 \leqslant \theta \leqslant 2\pi$

37–38 ▪ Use a calculator to find the length of the curve correct to four decimal places.

37. $r = 3 \sin 2\theta$

38. $r = 4 \sin 3\theta$

9.5 CONIC SECTIONS IN POLAR COORDINATES

In your previous study of conic sections, parabolas were defined in terms of a focus and directrix whereas ellipses and hyperbolas were defined in terms of two foci. After reviewing those definitions and equations, we present a more unified treatment of all three types of conic sections in terms of a focus and directrix. Furthermore, if we place the focus at the origin, then a conic section has a simple polar equation. In Chapter 10 we will use the polar equation of an ellipse to derive Kepler's laws of planetary motion.

CONICS IN CARTESIAN COORDINATES

Here we provide a brief reminder of what you need to know about conic sections. A more thorough review can be found on the website www.stewartcalculus.com.

Recall that a **parabola** is the set of points in a plane that are equidistant from a fixed point F (called the **focus**) and a fixed line (called the **directrix**). This definition is illustrated by Figure 1. Notice that the point halfway between the focus and the directrix lies on the parabola; it is called the **vertex**. The line through the focus perpendicular to the directrix is called the **axis** of the parabola.

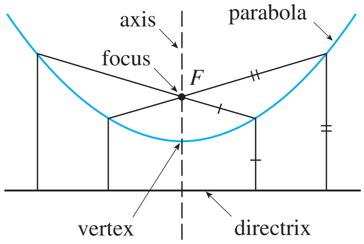
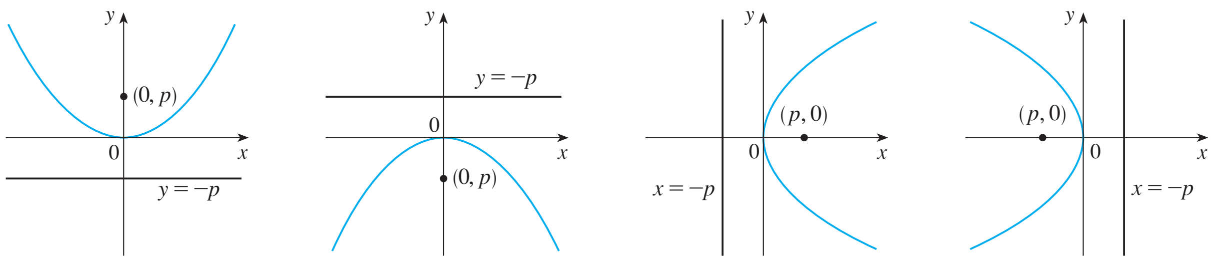


FIGURE 1

A parabola has a very simple equation if its vertex is placed at the origin and its directrix is parallel to the x-axis or y-axis. If the focus is on the y-axis at the point $(0, p)$, then the directrix has the equation $y = -p$ and an equation of the parabola is $x^2 = 4py$. [See parts (a) and (b) of Figure 2.] If the focus is on the x-axis at $(p, 0)$, then the directrix is $x = -p$ and an equation is $y^2 = 4px$ as in parts (c) and (d).



(a) $x^2 = 4py,\ p > 0$ (b) $x^2 = 4py,\ p < 0$ (c) $y^2 = 4px,\ p > 0$ (d) $y^2 = 4px,\ p < 0$

FIGURE 2