Name: *ANSWER KEY* 6/10/2014

Math 202 Calculus II Quiz 1

Please show all work and box your final answers. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Find $(f^{-1})'(a)$, where $f(x) = 5x^3 + 5x + 12$ and $a = 12$.

$f(x) = 5x^3 + 5x + 12 = 12$

$5x(x^2+1) = 0$

$x = 0$

$\therefore f^{-1}(12) = 0$

$\Rightarrow (f^{-1})'(12) = \dfrac{1}{f'(f^{-1}(12))}$

INVERSE FUNCTION THEOREM

$\left.\begin{array}{l} f'(x) = 15x^2 + 5 \\ f'(0) = 5 \end{array}\right\} \longrightarrow = \dfrac{1}{f'(0)} = \boxed{\dfrac{1}{5}}$

2. Suppose a sample of a radioactive material which is known to decay exponentially is placed on a scale. Initially, the scale reads 90 grams, and 10 days later it reads 80 grams.

(a) Find a function $M(t)$ which gives the mass of the sample after t days.

$M(t) = M(0)e^{kt} = 90e^{kt}$

$M(10) = 90e^{10k} = 80$

$e^{10k} = \frac{8}{9}$

$10k = \ln\left(\frac{8}{9}\right)$

$k = \frac{1}{10}\ln\left(\frac{8}{9}\right)$

$\Rightarrow M(t) = 90e^{\ln\left(\frac{8}{9}\right)\frac{t}{10}}$

$\boxed{M(t) = 90\left(\frac{8}{9}\right)^{t/10}}$

(b) Find the *half-life* of the material.

SOLVE FOR t: $M(t) = 90\left(\frac{8}{9}\right)^{t/10} = 45$

$\left(\frac{8}{9}\right)^{t/10} = \frac{1}{2}$

$\frac{t}{10}\ln\left(\frac{8}{9}\right) = \ln\left(\frac{1}{2}\right)$

$\boxed{t = \dfrac{10\ln\left(\frac{1}{2}\right)}{\ln\left(\frac{8}{9}\right)}}$

3. Differentiate the following functions.

(a) $y = \ln \dfrac{1+2x}{3-4x}$

$$y = \ln(1+2x) - \ln(3-4x)$$

$$y' = \frac{1}{1+2x}(2) - \frac{1}{3-4x}(-4)$$

ALSO GOOD

$$\boxed{y' = \frac{2}{1+2x} + \frac{4}{3-4x}} \left(= \frac{10}{(1+2x)(3-4x)} = \frac{10}{3+2x-8x^2} \right)$$

(b) $y = 9^x$

$$\left(y = \left(e^{\ln 9}\right)^x = e^{\ln(9)x} \right)$$

$$\boxed{y' = 9^x \ln(9)}$$

(c) $y = x^{\sin x}$

$$y = \left(e^{\ln x}\right)^{\sin x} = e^{\ln(x)\sin(x)}$$

$$y' = e^{\ln(x)\sin(x)}\left(\frac{1}{x}\sin(x) + \ln(x)\cos(x)\right)$$

$$\boxed{y' = x^{\sin(x)}\left(\frac{\sin(x)}{x} + \ln(x)\cos(x)\right)}$$

4. Evaluate the following integrals.

(a) $\displaystyle\int \frac{e^x+2}{3e^x}\,dx$

$$= \int \frac{e^x}{3e^x}\,dx + \int \frac{2}{3e^x}\,dx$$

$$= \frac{1}{3}\int dx + \frac{2}{3}\int e^{-x}\,dx$$

$$= \boxed{\frac{1}{3}x - \frac{2}{3}e^{-x} + C}$$

(b) $\displaystyle\int_2^4 \frac{2^{x-1}}{2^{x-1}+1}\,dx$

LET $u = 2^{x-1} + 1$

$du = 2^{x-1}\,\mathrm{Ln}(2)\,dx$

$\frac{1}{\mathrm{Ln}(2)}\,du = 2^{x-1}\,dx$

$$\frac{1}{\mathrm{Ln}(2)}\int_3^9 \frac{1}{u}\,du = \left.\frac{\mathrm{Ln}|u|}{\mathrm{Ln}(2)}\right|_3^9$$

$$= \boxed{\frac{\mathrm{Ln}(9) - \mathrm{Ln}(3)}{\mathrm{Ln}(2)} = \frac{\mathrm{Ln}(3)}{\mathrm{Ln}(2)} = \mathrm{Log}_2(3)}$$

ALL GOOD