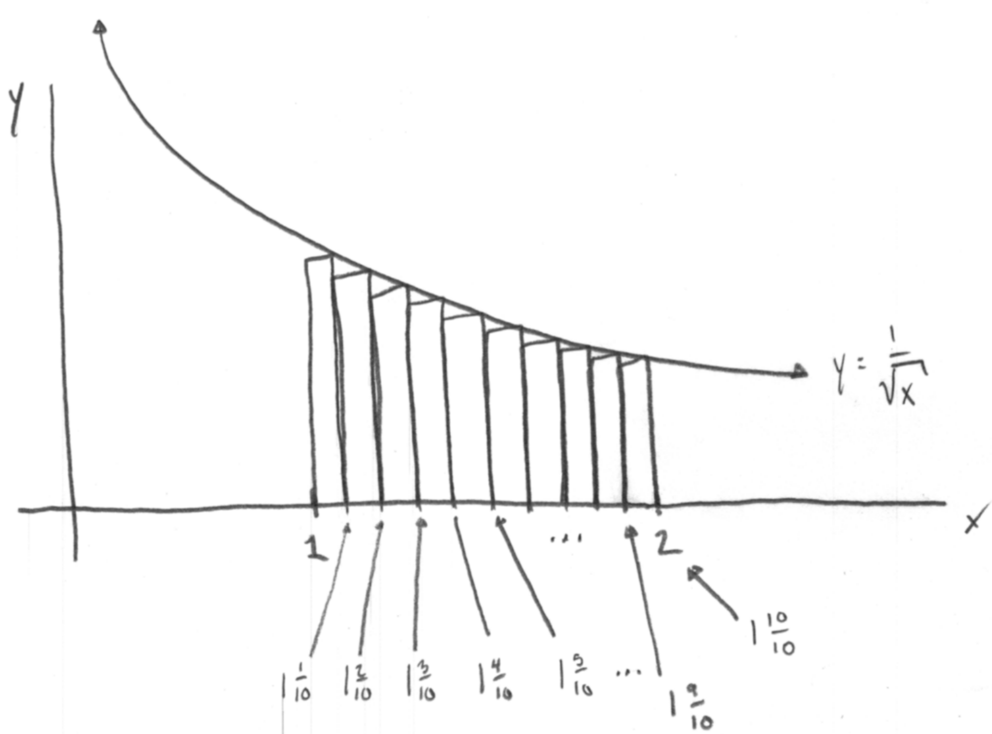
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{\sqrt{n}\sqrt{n+i}} = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\frac{1}{\sqrt{1+\frac{i}{n}}}}_{f(1+i\Delta x)} \underbrace{\frac{1}{n}}_{\Delta x = \frac{2-1}{n} = \frac{1}{n}}$$

WHERE $f(x) = \frac{1}{\sqrt{x}}$

$$= \int_1^2 \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}\Big|_1^2 = \boxed{2(\sqrt{2}-1) \approx 0.8284}$$



HERE $n = 10$.

TAKE LIMIT AS $n \to \infty$. $= \int_1^2 \frac{1}{\sqrt{x}}\,dx$!

$$\int x^3 \sqrt{1-x^2}^{\,5}\,dx \qquad \text{LET} \quad u = 1 - x^2 \longrightarrow x^2 = 1 - u$$

$$du = -2x\,dx$$

$$-\tfrac{1}{2}\,du = x\,dx$$

$$\int x^2 (1-x^2)^{5/2}\, x\,dx \rightsquigarrow -\frac{1}{2}\int (1-u)\,u^{5/2}\,du$$

$$= -\frac{1}{2}\int u^{5/2} - u^{7/2}\,du = -\frac{1}{2}\left(\frac{2}{7}u^{7/2} - \frac{2}{9}u^{9/2}\right) + C$$

$$\rightsquigarrow \boxed{\frac{1}{9}(1-x^2)^{9/2} - \frac{1}{7}(1-x^2)^{7/2} + C}$$

$$\int x^3 \sqrt{1-x^2}^{\,5}\,dx \qquad \text{LET} \quad x = \sin\theta$$

$$dx = \cos\theta\,d\theta$$

$$\rightsquigarrow \int \sin^3\theta \sqrt{1-\sin^2\theta}^{\,5} \cos\theta\,d\theta = \int \sin^3\theta \cos^6\theta\,d\theta$$

$$= \int (1-\cos^2\theta)\cos^6\theta \sin\theta\,d\theta \qquad \text{LET} \quad u = \cos\theta \left(= \sqrt{1-\sin^2\theta} = (1-x^2)^{1/2}\right)$$

$$du = -\sin\theta\,d\theta$$

$$\rightsquigarrow -\int (1-u^2)u^6\,du = \frac{1}{9}u^9 - \frac{1}{7}u^7 + C$$

SAME!!