Name: *Answer Key* 3/15/2018

Math 202 Calculus II **Quiz 2**

All electronic devices must be turned off and put away (e.g. cellphones, calculators, etc.). Keep your eyes on your own paper and do not talk to other students. Answer all of the following questions. Show all of your work and put boxes around your final answers. If you need more room, you may continue your work on the backs of the pages. Good luck!

1. (18 points) Evaluate the following limits.

(a) $\lim_{x\to 0} \dfrac{\ln(\sec x)}{x^2}$: $\frac{0}{0}$

$$\overset{L'HO}{=} \lim_{x\to 0} \frac{\frac{1}{\sec x}\sec x \tan x}{2x} = \lim_{x\to 0} \frac{\tan x}{2x} : \frac{0}{0}$$

$$\overset{L'HO}{=} \lim_{x\to 0} \frac{\sec^2 x}{2} = \boxed{\frac{1}{2}}$$

(b) $\lim_{x\to\infty} (2^x + x)^{1/x}$: ∞^0

$$= \lim_{x\to\infty} \mathrm{EXP}\left[\ln\left((2^x + x)^{1/x}\right)\right]$$

$$= \mathrm{EXP}\left[\lim_{x\to\infty} \frac{\ln(2^x + x)}{x} : \frac{\infty}{\infty}\right]$$

$$\overset{L'HO}{=} \mathrm{EXP}\left[\lim_{x\to\infty} \frac{2^x \ln 2 + 1}{2^x + x} : \frac{\infty}{\infty}\right]$$

$$\overset{L'HO}{=} \mathrm{EXP}\left[\lim_{x\to\infty} \frac{2^x (\ln 2)^2}{2^x \ln 2 + 1} : \frac{\infty}{\infty}\right]$$

$$\overset{L'HO}{=} \mathrm{EXP}\left[x\to\infty \ \frac{2^x (\ln 2)^3}{2^x (\ln 2)^2}\right] = \mathrm{EXP}\left[\ln 2\right] = \boxed{2}$$

(c) $\lim_{x\to\infty} \frac{\sinh x}{e^x}$ $= \lim_{x\to\infty} \frac{e^x - e^{-x}}{2e^x}$

$= \lim_{x\to\infty} \frac{e^x(1-e^{-2x})}{2e^x} = \lim_{x\to\infty} \frac{1-e^{-2x}}{2}$

$= \boxed{\frac{1}{2}}$

2. (18 points) Evaluate the following integrals.

(a) $\int_{-\pi}^{\pi} x^2 \cos x \, dx$ $= 2\int_0^{\pi} x^2 \cos x \, dx$ SINCE THE INTEGRAND IS EVEN

LET $u = x^2$ $\quad v = \sin x$

$du = 2x\,dx$ $\quad dv = \cos x \, dx$

$= \underbrace{2x^2 \sin x \Big|_0^{\pi}}_{0} - 4\int_0^{\pi} x \sin x \, dx$ LET $u = x$ $\quad v = -\cos x$

$du = dx$ $\quad dv = \sin x \, dx$

$= -4\left[-x\cos x \Big|_0^{\pi} + \underbrace{\int_0^{\pi} \cos x \, dx}_{0} \right] = -4\left[-\pi(-1)\right]$

$= \boxed{-4\pi}$

(b) $\displaystyle\int_0^{1/2} \sin^{-1} x \, dx$

LET $u = \sin^{-1} x$ $\qquad v = x$

$du = \dfrac{1}{\sqrt{1-x^2}}\, dx \qquad dv = dx$

$$= x \sin^{-1} x \Big|_0^{1/2} - \int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\, dx$$

LET $u = 1 - x^2$

$-\frac{1}{2}\, du = x\, dx$

$$= \left(\frac{1}{2} \cdot \frac{\pi}{6} - 0 \right) + \frac{1}{2} \int_1^{3/4} u^{-1/2}\, du$$

$$= \frac{\pi}{12} + \Big[\sqrt{u} \Big]_1^{3/4} = \boxed{\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 \quad \text{OR} \quad = \frac{\pi + 6\sqrt{3} - 12}{12}}$$

(c) $\displaystyle\int \sin^2 x \cos^5 x \, dx \;=\; \int \sin^2 x \,(1 - \sin^2 x)^2 \cos x \; dx$

LET $u = \sin x$

$du = \cos x \, dx$ $\quad \rightsquigarrow \quad \displaystyle\int u^2 (1-u^2)^2 \, du$

$$= \int u^2 - 2u^4 + u^6 \, du = \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$$

$$\rightsquigarrow \boxed{\frac{1}{3}\sin^3 x - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C}$$

3. (7 points) At what point of the curve $y = \cosh x$ does the tangent line have slope 1?

$\frac{dy}{dx} = \text{SINH}\, X$. SOLVE : $\text{SINH}\ X = 1$

$$\frac{e^x - e^{-x}}{2} = 1 \rightarrow \left[e^x - 2 - e^{-x} = 0 \right] e^x$$

$$(e^x)^2 - 2(e^x) - 1 = 0$$

QUAD. FORMULA : $e^x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

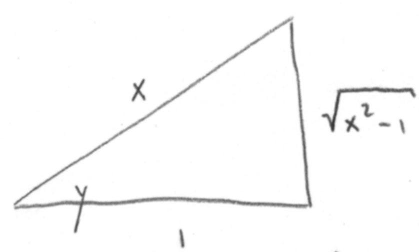
SINCE $e^x > 0$ ALWAYS, $e^x = 1 + \sqrt{2}$

$\Rightarrow \boxed{x = \ln(1+\sqrt{2})}$

$\boxed{\text{POINT : } \left(\ln(1+\sqrt{2}),\ \cosh(1+\sqrt{2})\right) = \left(\ln(1+\sqrt{2}),\ \frac{\sqrt{2}}{1+\sqrt{2}}\right)}$

4. (7 points) Prove that $\frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$.

LET $y = \sec^{-1} x$. FIND $\frac{dy}{dx}$.



$\sec y = x$

$\sec y \tan y \frac{dy}{dx} = 1 \rightarrow \frac{dy}{dx} = \frac{1}{\sec y \tan y}$

$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$ ✓