

Please show all work and box your final answers. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Let  $f$  be the one-to-one function  $f(x) = 2x^3 + 3x^2 + 7x + 4$ . Find  $(f^{-1})'(4)$ .

2. State the domain of the function  $f(x) = \frac{e^{x-1}}{(\ln x)(1 - \ln x)}$ .

3. For each of the following, find  $\frac{dy}{dx}$ .

(a)  $y = x^{\sqrt{x}}(x^{\ln x})$

(b)  $y = e^{\arctan^2 x}$

(c)  $y = \frac{5^x \sqrt[4]{x^3}}{x^7(x+1)^2}$

4. Suppose a colony of bacteria grows according to the law of *natural decay*. Initially, the colony contains 2400 bacteria, and 3 days later the colony contains 6000 bacteria. How long will it take the population to grow from 2400 bacteria to 9600 bacteria?

5. Derive the derivative  $\frac{d}{dx} [\sec^{-1} x]$ .

6. Evaluate the following limits.

(a)  $\lim_{x \rightarrow \infty} \frac{\cosh(1/x)}{e^{1/x}}$

(b)  $\lim_{x \rightarrow 0} \frac{5^x - 4^x}{3^x - 2^x}$

(c)  $\lim_{x \rightarrow 0^+} (\cos x)^{1/x^2}$

7. Evaluate the following integrals.

(a)  $\int x^2 e^x dx$

(b)  $\int \tan^5 \pi\theta d\theta$

8. Evaluate the following integrals.

(a)  $\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}} dx$

(b)  $\int \frac{x^2 + 2x - 1}{x^3 - x} dx$