

Please show all work and box your final answers. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. A small (12 oz.) Starbucks coffee contains 235 mg of caffeine. The half-life of caffeine in an adult human is approximately 5.5 hours. (The kidneys are responsible for filtering out the caffeine according to the *law of natural decay*.) How many hours after drinking one small Starbucks coffee does an adult human have only 40mg of caffeine remaining in their body?

2. (a) Given $f(x) = \sin^{-1}(x)$, state the domain and range of f . Also state $f'(x)$.

- (b) Given $g(x) = \cos^{-1}(x)$, state the domain and range of g . Also state $g'(x)$.

- (c) Given $h(x) = \tan^{-1}(x)$, state the domain and range of h . Also state $h'(x)$.

3. Evaluate the following expressions.

(a) $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$

(b) $\cos^{-1}\left(\cos \frac{-\pi}{6}\right)$

(c) $\tan^{-1}(\tan 0)$

4. Simplify the expression $\cot^2(\sin^{-1} x)$.

5. Differentiate $y = \sin^{-1}(\sqrt{1-x})$. Simplify your answer.

6. Prove that $\frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}$. *Hint: The following identities are true for all real t :*

$$\cosh^2 t - \sinh^2 t = 1, \quad 1 - \tanh^2 t = \operatorname{sech}^2 t, \quad \coth^2 t - 1 = \operatorname{csch}^2 t.$$