Name: * ANSWER KEY * **6/26/2018**

Math 202 Calculus II **Quiz 3**

Please show all work and box your final answers. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Circle the indeterminate forms which appear below, and do not circle the others.

$$\frac{0}{\infty} \quad \frac{0}{0} \quad \frac{\infty}{0} \quad \frac{\infty}{\infty}$$

$$0^0 \quad 0^1 \quad 0^\infty \quad 1^0 \quad 1^1 \quad 1^\infty \quad \infty^0 \quad \infty^1 \quad \infty^\infty$$

2. Evaluate the following limit. If you use L'hospital's rule, you must first indicate which indeterminate form the limit has.

$$\lim_{x\to\infty}\left(1+\frac{1}{ax}\right)^{bx}$$

IND. FORM 1^∞.

$$= \mathrm{EXP}\left[\ln\left(\lim_{x\to\infty}\left(1+\frac{1}{ax}\right)^{bx}\right)\right]$$

$$= \mathrm{EXP}\left[\lim_{x\to\infty}\ln\left(\left(1+\frac{1}{ax}\right)^{bx}\right)\right]$$

$$= \mathrm{EXP}\left[\lim_{x\to\infty} bx\ln\left(1+\frac{1}{ax}\right)\right] \quad \text{IND. FORM } \infty\cdot 0$$

$$= \mathrm{EXP}\left[\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{ax}\right)}{\frac{1}{bx}}\right] \quad \text{IND FORM } \frac{0}{0}$$

L'HÔ RULE $\rightarrow$ $$\mathrm{EXP}\left[\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{ax}}\left(\frac{\cancel{-1}}{a\cancel{x^2}}\right)}{\frac{\cancel{-1}}{b\cancel{x^2}}}\right] = \mathrm{EXP}\left[\lim_{x\to\infty}\frac{1}{1+\underbrace{\frac{1}{ax}}_{\to 0}}\cdot\frac{b}{a}\right]$$

$$= \mathrm{EXP}\left[\frac{b}{a}\right] = \boxed{e^{b/a}}$$

3. Evaluate the following limits. If you use L'hospital's rule, you must first indicate which indeterminate form the limit has.

(a) $\lim_{x\to 0} \frac{\cos(mx) - \cos(nx)}{x^2}$ IND, FORM $\frac{0}{0}$

L'HÔ RULE $\longrightarrow$ $\lim_{x\to 0} \frac{-m\sin(mx) + n\sin(nx)}{2x}$ IND. FORM $\frac{0}{0}$

L'HÔ RULE $\longrightarrow$ $\lim_{x\to 0} \frac{-m^2\cos(mx) + n^2\cos(nx)}{2}$

$= \boxed{\frac{n^2 - m^2}{2}}$

(b) $\lim_{x\to 0} \frac{1 - e^{-2x}}{\sec x}$ $= \frac{\lim_{x\to 0}(1 - e^{-2x})}{\lim_{x\to 0}(\sec x)} = \frac{0}{1} = \boxed{0}$

4. Evaluate the following integrals.

(a) $\int_1^4 \sqrt{x}\ln x\,dx$ LET $u = \ln x$ $\quad v = \frac{2}{3}x^{3/2}$

$du = \frac{1}{x}dx \quad dv = \sqrt{x}\,dx$

$$= uv\Big|_1^4 - \int_1^4 v\,du = \frac{2}{3}x^{3/2}\ln x\Big|_1^4 - \int \frac{2}{3}x^{3/2}x^{-1}\,dx$$

$$= \frac{2}{3}4^{3/2}\ln 4 - 0 - \frac{2}{3}\int_1^4 x^{1/2}\,dx = \frac{2}{3}(4^{1/2})^3\ln 4 - \frac{4}{9}x^{3/2}\Big|_1^4$$

$$= \frac{16\ln 4}{3} - \frac{4}{9}(8-1) = \frac{16\ln 4}{3} - \frac{28}{9} \text{ or } \boxed{\frac{48\ln 4 - 28}{9}}$$

(b) $\int e^{-\theta}\cos 2\theta\,d\theta$ LET $u = \cos 2\theta$ $\quad v = -e^{-\theta}$

$du = -2\sin 2\theta\,d\theta \quad dv = e^{-\theta}\,d\theta$

$$= uv - \int v\,du = -e^{-\theta}\cos 2\theta - 2\int e^{-\theta}\sin 2\theta\,d\theta$$

$u = \sin 2\theta \quad v = -e^{-\theta}$

$du = 2\cos 2\theta\,d\theta \quad dv = e^{-\theta}\,d\theta$

$$= -e^{-\theta}\cos 2\theta - 2\left[uv - v\,du\right] = -e^{-\theta}\cos 2\theta - 2\left[-e^{-\theta}\sin 2\theta + 2\int e^{-\theta}\cos 2\theta\,d\theta\right]$$

$$\therefore \int e^{-\theta}\cos 2\theta\,d\theta = -e^{-\theta}\cos 2\theta + 2e^{-\theta}\sin 2\theta - 4\int e^{-\theta}\cos 2\theta\,d\theta$$

$$5\int e^{-\theta}\cos 2\theta\,d\theta = e^{-\theta}(2\sin 2\theta - \cos 2\theta)$$

$$\int e^{-\theta}\cos 2\theta\,d\theta = \boxed{\frac{1}{5}e^{-\theta}(2\sin 2\theta - \cos 2\theta) + C}$$

5. Evaluate the following integrals.

(a) $\int_0^{\pi/4} \sec^6\theta \tan^4\theta \, d\theta \quad = \int_0^{\pi/4} \sec^4\theta \tan^4\theta \sec^2\theta \, d\theta$

$$= \int_0^{\pi/4} (\tan^2\theta + 1)^2 \tan^4\theta \sec^2\theta \, d\theta$$

LET $u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$$\rightsquigarrow \int_0^1 (u^2+1)^2 u^4 \, du = \int_0^1 u^8 + 2u^6 + u^4 \, du$$

$$= \frac{1}{9}u^9 + \frac{2}{7}u^7 + \frac{1}{5}u^5 \Big|_0^1 = \boxed{\frac{1}{9} + \frac{2}{7} + \frac{1}{5}} = \frac{35 + 90 + 63}{315}$$

$$= \frac{188}{315}$$

(b) $\int \sin^4 2\theta \, d\theta \quad = \int (\sin^2\theta)^2 \, d\theta = \int \left(\frac{1}{2}(1 - \cos 2\theta)\right)^2$

$$= \frac{1}{4}\int 1 - 2\cos 2\theta + \underbrace{\cos^2 2\theta}_{\frac{1}{2}(1+\cos 4\theta) = \frac{1}{2} + \frac{1}{2}\cos 4\theta} \, d\theta$$

$$= \frac{1}{4}\int \frac{3}{2} - 2\cos 2\theta + \frac{1}{2}\cos 4\theta \, d\theta$$

$$= \frac{1}{4}\left[\frac{3}{2}\theta - \sin 2\theta + \frac{1}{8}\sin 4\theta\right] + C \quad \text{OR} \quad \frac{12\theta - 8\sin 2\theta + \sin 4\theta}{32} + C$$

OR $\boxed{\frac{3}{8}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta + C}$