§10.1 THREE-DIMENSIONAL COORDINATE SYSTEMS

JOHN ADAMSKI
8/25/2014

3. DISTANCE TO YZ-PLANE IS THE ABSOLUTE VALUE OF THE X-COORDINATE.

$\therefore$ $\boxed{C(2,4,6)}$ IS CLOSEST TO YZ-PLANE.

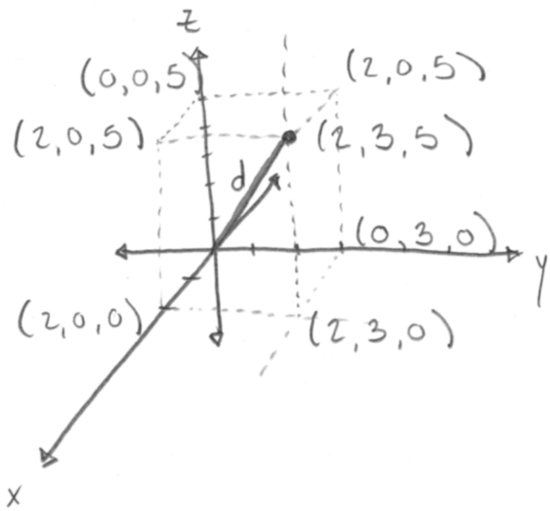
A POINT LIES IN THE XZ-PLANE $\iff$ ITS y-COORD. IS 0.

$\therefore$ $\boxed{A(-4,0,-1)}$ IS IN THE XZ-PLANE.

4. $(2,3,5) \xrightarrow[z=0]{xy} \boxed{(2,3,0)}$

$\xrightarrow[x=0]{yz} \boxed{(0,3,5)}$

$\xrightarrow[y=0]{xz} \boxed{(2,0,5)}$



$$d = \sqrt{2^2+3^2+5^2} = \boxed{\sqrt{38}}$$

7. (a)

$$|PQ| = \sqrt{(3-7)^2+(-2-0)^2+(-3-1)^2} = \sqrt{4^2+2^2+4^2} = 6$$

$$|QR| = \sqrt{(7-1)^2+(0-2)^2+(1-1)^2} = \sqrt{6^2+2^2} = \sqrt{40} = 2\sqrt{10}$$

$$|PR| = \sqrt{(3-1)^2+(-2-2)^2+(-3-1)^2} = \sqrt{2^2+4^2+4^2} = 6$$

THIS TRIANGLE IS $\boxed{\text{ISOSCELES}}$ ✓

BUT $6^2+6^2 \neq 40$, SO IT IS NOT A RIGHT TRIANGLE

(b)

$$|PQ| = \sqrt{2^2+2^2+1^2} = 3$$

$$|QR| = \sqrt{0^2+6^2+3^2} = \sqrt{45} = 3\sqrt{5}$$

$$|PR| = \sqrt{2^2+4^2+4^2} = 6$$

THIS TRIANGLE IS NOT ISOSCELES, BUT IS A $\boxed{\text{RIGHT TRIANGLE}}$ SINCE $3^2+6^2 = 45$.

10. EQ: $(x-2)^2+(y+6)^2+(z-4)^2 = 25$

INTERSECTED WITH XY-PLANE $\longrightarrow$ SET $z=0$

$$(x-2)^2+(y+6)^2+(0-4)^2=25$$

$$(x-2)^2+(y+6)^2 = 9 \leftarrow \boxed{\text{CIRCLE OF RADIUS } 3}$$

INTERSECTED WITH XZ-PLANE $\longrightarrow$ SET $y=0$

$$(x-2)^2+(0+6)^2+(z-4)^2 = 25$$

$$(x-2)^2+(z-4)^2 = -11 \leftarrow$$

NO SOLUTIONS

(THE SPHERE DOES NOT INTERSECT THE XZ-PLANE)

INTERSECTED WITH YZ-PLANE $\longrightarrow$ SET $x=0$

$$(0-2)^2+(y+6)^2+(z-4)^2 = 25$$

$$(y+6)^2+(z-4)^2 = 21 \leftarrow \boxed{\text{CIRCLE OF RADIUS } \sqrt{21}}$$

11. RADIUS r = DISTANCE FROM CENTER TO POINT ON SPHERE

$$= \sqrt{1^2+5^2+2^2} = \sqrt{30}$$

$\therefore$ SPHERE : $\boxed{(x-3)^2+(y-8)^2+(z-1)^2 = 30}$

13. $x^2-2x\ (+1) + y^2-4y\ (+4) + z^2+8z\ (+16) + 15\ (+1)(+4)(+16)$

$$(x-1)^2+(y-2)^2+(z+4)^2 = 36$$

THIS IS A SPHERE WITH $\boxed{\text{CENTER } (1,2,-4) \text{ \& RADIUS } 6.}$

15. $2x^2 - 8x + 2y^2 + 2z^2 + 24z = 1$

$$x^2 - 4x + y^2 + z^2 + 12z = \frac{1}{2}$$

$$(x-2)^2 + y^2 + (z+6)^2 = \frac{1}{2} + 4 + 36 = \frac{81}{2}$$

SPHERE WITH $\boxed{\text{CENTER } (2,0,-6) \text{ \& RADIUS } \frac{9}{\sqrt{2}}}$

19. (a) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 36$

(b) " " " $= 4$

(c) " " " $= 9$

23. HALF-SPACE LEFT OF PLANE $y = 8$

26. $z^2 = 1 \Rightarrow z = \pm 1$

BOTH $\underbrace{\text{HORIZONTAL PLANES}}$ $z = 1$ & $z = -1$

i.e. $\parallel$ TO xy-PLANE

29. INFINITELY TALL CYLINDER OF RADIUS 3, CENTERED ON z-AXIS.

30. $\longrightarrow$ $x^2 + y^2 + (z-1)^2 > 1$

ALL POINTS OUTSIDE OF THE BALL OF RADIUS 1 CENTERED AT $(0,0,1)$

32. $\boxed{x^2 + y^2 \le 4 \text{ AND } 0 \le z \le 8}$

SOLID CYLINDER

RESTRICTED TO LIE BETWEEN THESE 2 PLANES $z=0$ (xy-PLANE), $z = 8$

34. $x^2 + y^2 + z^2 = 4 \quad , \quad z \geq 0$

35. $|XA| = |XB| \leftarrow$ X IS EQUIDISTANT FROM A & B

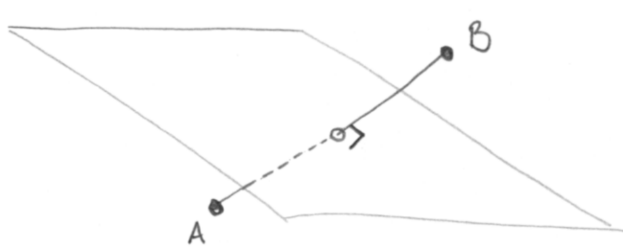
$$\sqrt{(x+1)^2 + (y-5)^2 + (z-3)^2} = \sqrt{(x-6)^2 + (y-2)^2 + (z+2)^2}$$

$$(x+1)^2 + (y-5)^2 + (z-3)^2 = (x-6)^2 + (y-2)^2 + (z+2)^2$$

$$\cancel{x^2} + 2x + 1 + \cancel{y^2} - 10y + 25 + \cancel{z^2} - 6z + 9 = \cancel{x^2} - 12x + 36 + \cancel{y^2} - 4y + 4$$
$$+ \cancel{z^2} + 4z + 4$$

$$\boxed{14x - 6y - 10z = 9}$$

THIS MUST BE THE PLANE THAT CUTS PERPINDICULARLY THE LINE SEGMENT CONNECTING THE TWO POINTS, AT ITS MIDPOINT.



37. SPHERES $x^2 + y^2 + z^2 = 4$ (RADIUS 2)

$(x-2)^2 + (y-2)^2 + (z-2)^2 = 1$ (RADIUS 1)

THE CENTERS ARE $\sqrt{2^2 + 2^2 + 2^2} = \sqrt{12} = 2\sqrt{3}$ APART.

$\therefore$ THE SPHERES ARE $2\sqrt{3} - 2 - 1 = \boxed{2\sqrt{3} - 3}$ APART