§ 10.4 CROSS PRODUCT

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1. $\langle 6,0,-2\rangle \times \langle 0,8,0\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 0 & -2 \\ 0 & 8 & 0 \end{vmatrix} = \langle 16, 0, 48\rangle$

$\perp$: $\langle 6,0,-2\rangle \cdot \langle 16,0,48\rangle = 6\cdot 16 - 2\cdot 48 = 0$ ✓

$\langle 0,8,0\rangle \cdot \langle 16,0,48\rangle = 0$ ✓

9. $(\hat{i}\times\hat{j})\times\hat{k} = \hat{k}\times\hat{k} = 0$

10. $\hat{k}\times(\hat{i}-2\hat{j}) = (\hat{k}\times\hat{i}) - 2(\hat{k}\times\hat{j}) = \hat{j} - 2(-\hat{i}) = \hat{j} + 2\hat{i}$

11. $(\hat{j}-\hat{k})\times(\hat{k}-\hat{i}) = (\hat{j}-\hat{k})\times\hat{k} - (\hat{j}-\hat{k})\times\hat{i}$

$= (\hat{j}\times\hat{k}) - (\hat{k}\times\hat{k}) - \left[(\hat{j}\times\hat{i}) - (\hat{k}\times\hat{i})\right]$

$= \hat{i} - 0 - \left[-\hat{k} - \hat{j}\right] = \hat{i} + \hat{k} + \hat{j}$

13. (a) VECTOR (b) NO. VECTOR × SCALAR ☹ (c) VECTOR

(d) NO. VECTOR · SCALAR ☹ (e) NO. SCALAR × SCALAR ☹

(f) SCALAR.

14. $|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta = 4\cdot 5\cdot\sin 45^\circ = 10\sqrt{2}$. OUT OF THE PAGE

17. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 4 & 2 & 1 \end{vmatrix} = -7\hat{i} + 10\hat{j} + 8\hat{k}$

OPPOSITES!

$\vec{b}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 7\hat{i} - 10\hat{j} - 8\hat{k}$

19. ORTHOGONAL: $\langle 3,2,1\rangle \times \langle -1,1,0\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix} = \langle -1,-1,5\rangle$

UNIT LENGTH: $|\langle -1,-1,5\rangle| = \sqrt{(-1)^2+(-1)^2+5^2} = 3\sqrt{3}$

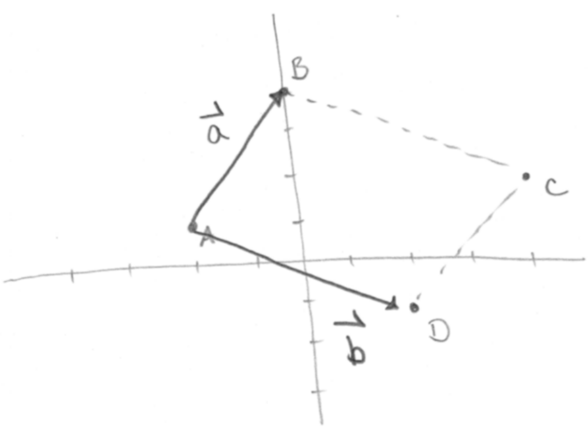
$\longrightarrow \frac{1}{3\sqrt{3}}\langle -1,-1,5\rangle$

22. NOTE THAT $(\vec{a}\times\vec{b}) \perp \vec{b}$

SO ANGLE BETWEEN $(\vec{a}\times\vec{b})$ & $\vec{b}$ IS $90°$.

$\Rightarrow (\vec{a}\times\vec{b})\cdot\vec{b} = |\vec{a}\times\vec{b}||\vec{b}|\underbrace{\cos 90°}_{0} = 0$ ✓

27.



$\vec{a} = \langle 2,3\rangle$

$\vec{b} = \langle 4,-2\rangle$

AREA $= |\vec{a}\times\vec{b}| = \left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 4 & -2 & 0 \end{vmatrix} \right\|$

$= |\langle 0,0,-16\rangle| = 16$

31. $\vec{PQ} = \langle 4,3,-2\rangle$, $\vec{PR} = \langle 5,5,1\rangle$

$\perp$ VECTOR: $\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & -2 \\ 5 & 5 & 1 \end{vmatrix} = \langle 13,-14,5\rangle$

AREA OF $\Delta = \frac{1}{2}|\vec{PQ}\times\vec{PR}| = \frac{1}{2}\sqrt{\underbrace{13^2+14^2+5^2}_{169+196+25}} = \frac{1}{2}\sqrt{390}$

33. $\left|(\vec{a}\times\vec{b})\cdot\vec{c}\right| = \left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ -1 & 1 & 2\end{vmatrix}\cdot\langle 2,1,4\rangle\right|$

$= \left|\langle 1,-5,3\rangle\cdot\langle 2,1,4\rangle\right| = |9| = 9$

35. $\vec{PQ} = \langle 4,2,2\rangle$, $\vec{PR} = \langle 3,3,-1\rangle$, $\vec{PS} = \langle 5,5,1\rangle$

$\text{Vol} = \left|(\langle 4,2,2\rangle\times\langle 3,3,-1\rangle)\cdot\langle 5,5,1\rangle\right|$

$= \left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 2 & 2\\ 3 & 3 & -1\end{vmatrix}\cdot\langle 5,5,1\rangle\right|$

$= \left|\langle -8,10,6\rangle\cdot\langle 5,5,1\rangle\right| = |-40+50+6| = 16$

38. $\left|(\vec{AB}\times\vec{AC})\cdot\vec{AD}\right| = \left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 4\\ 4 & -1 & -2\end{vmatrix}\cdot\langle 2,3,-6\rangle\right|$

$= \left|\langle 12,20,14\rangle\cdot\langle 2,3,-6\rangle\right| = |24+60-84| = 0$

SINCE THE VOLUME OF THE PARALLELOPIPED IS 0,

$\Rightarrow$ THE VECTORS $\vec{AB}$, $\vec{AC}$, $\vec{AD}$ MUST BE COPLANAR

$\Rightarrow$ THE POINTS A, B, C, D MUST BE COPLANAR.

39. $|\vec{\tau}| = |\vec{r}\times\vec{F}| = |\vec{r}||\vec{F}|\sin\theta$

$= (.18)(60)\sin(80^\circ) = 10.8\sin(80^\circ)$

41. LET'S USE UNIT VECTOR $\langle 0, \frac{3}{5}, -\frac{4}{5} \rangle$ FOR DIRECTION OF FORCE

$|\tau| = |\vec{r} \times \vec{F}|$ GIVES:

$$100 = \left| \langle 0, .3, 0 \rangle \times \lambda \langle 0, \overset{.6}{\tfrac{3}{5}}, \overset{-.8}{-\tfrac{4}{5}} \rangle \right|$$

WHERE $\lambda = |\vec{F}|$ (MAGNITUDE)

$$100 = \lambda \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & .3 & 0 \\ 0 & .6 & -.8 \end{matrix} \right\| = \lambda \sqrt{.24^2 + 0^2 + 0^2}$$

$$100 = .24\lambda \longrightarrow \lambda = \boxed{416\tfrac{2}{3}\ N}$$

43. $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = \sqrt{3}$

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta = 3$

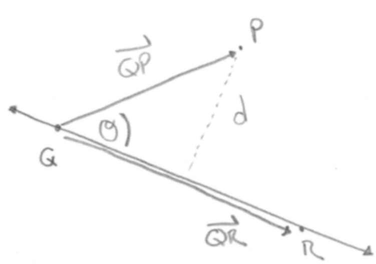
$\sqrt{1^2 + 2^2 + 2^2}$

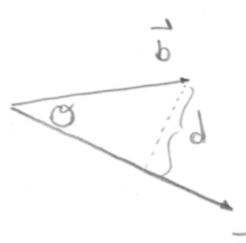
$$\Rightarrow \frac{|\vec{a}||\vec{b}| \sin\theta}{|\vec{a}||\vec{b}| \cos\theta} = \frac{3}{\sqrt{3}}$$

$\tan\theta = \sqrt{3}$

$$\boxed{\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}}$$

45. (a)





$d = |\vec{b}| \sin\theta$

$$d = \frac{|\vec{a}||\vec{b}| \sin\theta}{|\vec{a}|}$$

$$d = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|}$$

(b) $d = \dfrac{|\vec{QR} \times \vec{QP}|}{|\vec{QR}|} = \dfrac{\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -1 \\ 1 & -5 & -7 \end{matrix} \right\|}{|\langle -1, -2, -1 \rangle|}$

$$= \frac{|\langle 9, -8, 7 \rangle|}{|\langle -1, -2, -1 \rangle|} = \frac{\sqrt{81 + 64 + 49}}{\sqrt{1 + 4 + 1}} = \sqrt{\frac{194}{6}} = \boxed{\sqrt{\frac{97}{3}}}$$