§10.5 EQUATIONS OF LINES & PLANES

LINES

LINE THROUGH $\vec{r}_0$ IN DIRECTION $\vec{v}$:

VECTOR EQUATION: $\vec{r} = \vec{r}_0 + t\vec{v}$, $-\infty < t < \infty$

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle$$

VECTORS ARE EQUAL $\Longleftrightarrow$ COMPONENTS ARE EQUAL

PARAMETRIC EQ'S: $\Rightarrow$

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

$\downarrow$ SOLVE FOR t

SYMMETRIC EQ'S:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

(ASSUMING $a, b, c \neq 0$)

IF ONE IS ZERO, WE STILL SOLVE FOR t IN REMAINING PARAM. EQ'S.

e.g. IF $a = 0$

$$x = x_0 \; ; \quad \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

EXAMPLE

THE LINE SEGMENT FROM $\vec{r}_0$ TO $\vec{r}_1$ IS GIVEN BY

$$\vec{r}(t) = (1 - t)\vec{r}_0 + t\vec{r}_1 , \quad 0 \leq t \leq 1$$

Note: $\vec{r}(0) = \vec{r}_0$, $\vec{r}(1) = \vec{r}_1$

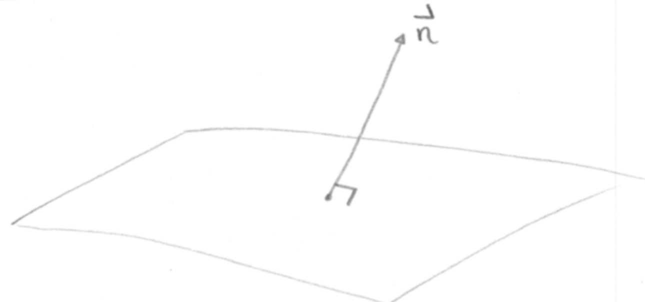
e.g. SHOW THAT LINES L_1 & L_2 ARE SKEW LINES (NOT ∥, DON'T INTERSECT)

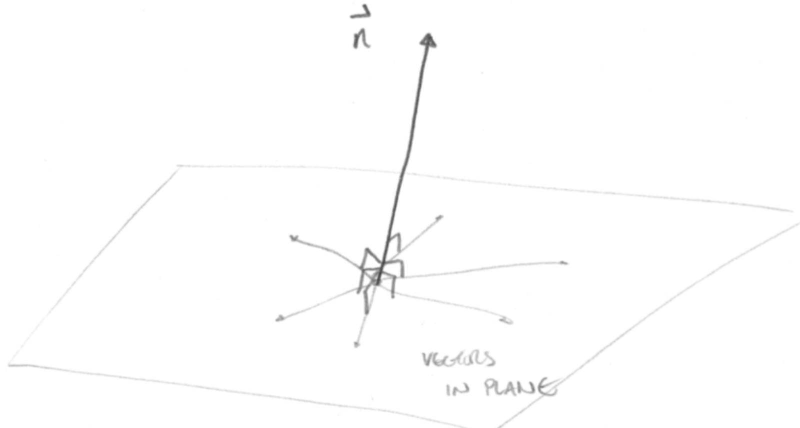
L_1: $x = 1 + t$, $y = -2 + 3t$, $z = 4 - t$

L_2: $x = 2s$, $y = 3 + s$, $z = -3 + 4s$

PLANES

NORMAL VECTOR $\vec{n}$



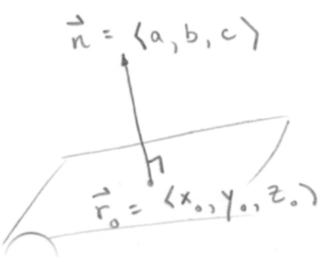


IF $\vec{r}_0$ IS IN THE PLANE & $\vec{n}$ IS NORMAL TO PLANE,

THEN $\vec{r}$ IS IN THE PLANE $\iff$ $\underbrace{\vec{r} - \vec{r}_0}_{\text{VECTOR IN THE PLANE}} \perp \vec{n}$

VECTOR EQ:

i.e. $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$ OR $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r}_0$



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SCALAR EQ:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$\to ax + by + cz + d = 0$$

EXAMPLES: POINT, NORMAL VECTOR | PLANE THROUGH 3 PNTS!

<u>NOTE:</u> THE ANGLE BETWEEN TWO PLANES IS DEFINED TO BE THE ACUTE ANGLE BETWEEN THEIR NORMAL VECTORS.

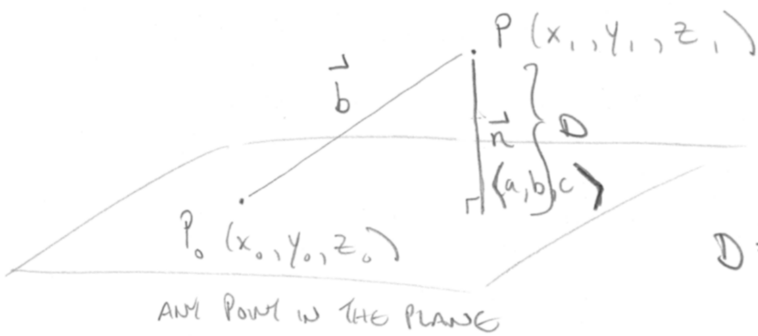
e.g. ‖ PLANES HAVE ‖ NORMAL VECTORS.

e.g. FIND ANGLE BETWEEN PLANES ....

$$\left(\text{RECALL} \quad \vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1||\vec{n}_2| \cos\theta \right)$$

EXAMPLE : p. 576 #6.b.

e.g. FIND DISTANCE FROM $P(x_1, y_1, z_1)$ TO PLANE $ax + by + cz + d = 0$.



$$D = \left| \text{comp}_{\vec{n}} \vec{b} \right| = \frac{|\vec{n} \cdot \vec{b}|}{|\vec{n}|}$$

$$D = \frac{|ax_1 + by_1 + cz_1|}{\sqrt{a^2 + b^2 + c^2}}$$

e.g. FIND DISTANCE BETWEEN 2 ‖ PLANES.