§11.2 LIMITS & CONTINUITY

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3. $\lim\limits_{(x,y)\to(1,2)} \left(5x^3 - x^2y^2\right) = 5(1)^3 - (1)^2(2)^2 = 5 - 4 = \boxed{1}$

POLYNOMIAL => CONTINUOUS EVERYWHERE (JUST PLUG IN!)

5. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 4y^2}{x^2 + 2y^2}$

IF $x=0$: $\lim\limits_{y\to 0} \dfrac{4y^2}{2y^2} = 2$

IF $y=0$: $\lim\limits_{x\to 0} \dfrac{x^4}{x^2} = 0$

DIFFERENT => LIMIT $\boxed{\text{D.N.E.}}$

7. $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^2 \sin^2 x}{x^4 + y^4}$

IF $x=0$: $\lim\limits_{y\to 0} \dfrac{y^2 \sin(0)}{y^4} = 0$

IF $y=x$: $\lim\limits_{x\to 0} \dfrac{x^2 \sin^2 x}{2x^4} = \lim\limits_{x\to 0} \dfrac{1}{2}\left(\dfrac{\sin x}{x}\right)^2 = \dfrac{1}{2}$

DIFFERENT => LIMIT $\boxed{\text{D.N.E.}}$

9. $$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$ LET $x = r\cos\theta$, $y = r\sin\theta$

AS $(x,y)\to(0,0)$, $r\longrightarrow 0$

$$= \lim_{r\to 0} \frac{r^2\cos\theta\sin\theta}{r} = \boxed{0}$$

11. $$\lim_{(x,y)\to(0,0)} \frac{x^2 y e^y}{x^4+4y^2}$$

IF $y=0$, $\displaystyle\lim_{x\to 0} \frac{0}{x^4} = 0$

IF $y=x^2$, $\displaystyle\lim_{x\to 0} \frac{x^4 e^{x^2}}{5x^4} = \frac{1}{5}$

DIFFERENT => LIMIT $\boxed{\text{D.N.E.}}$

13. $$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}\cdot\frac{\sqrt{x^2+y^2+1}+1}{\sqrt{x^2+y^2+1}+1}$$

$$= \lim_{(x,y)\to(0,0)} \frac{\cancel{(x^2+y^2)}\left(\sqrt{x^2+y^2+1}+1\right)}{\cancel{(x^2+y^2)}} = \boxed{2}$$

15. $$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$$

IF $x = y = 0$: $\lim_{z \to 0} \frac{0}{z^4} = 0$

IF $y = 0$, $x = z^2$: $\lim_{z \to 0} \frac{z^4}{2z^4} = \frac{1}{2}$

DIFFERENT $\Rightarrow$ LIMIT $\boxed{\text{D.N.E.}}$

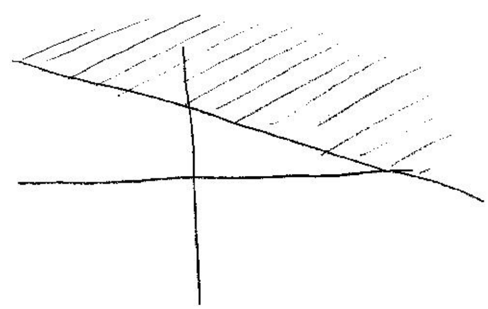
19. $h(x,y) = g(f(x,y)) = f(x,y)^2 + \sqrt{f(x,y)}$

$$= \boxed{(2x+3y-6)^2 + \sqrt{2x+3y-6}}$$

CONTINUOUS ON DOMAIN: $2x + 3y - 6 \geq 0$

$3y \geq -2x + 6$

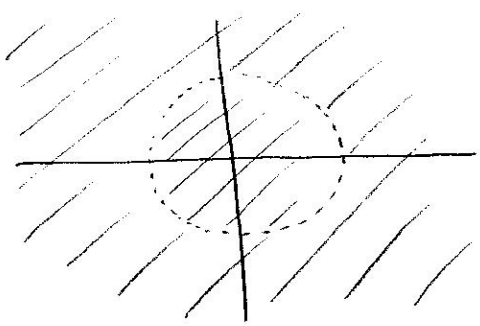
$y \geq -\frac{2}{3}x + 2$



21. CONTINUOUS ON DOMAIN: $1 - x^2 - y^2 \neq 0$

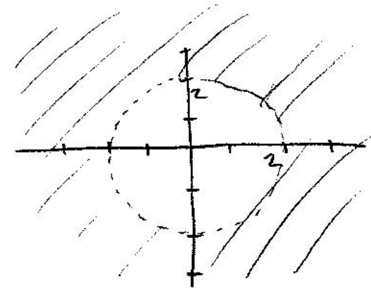
$1 \neq x^2 + y^2$

$\boxed{\text{EVERYWHERE EXCEPT UNIT CIRCLE}}$



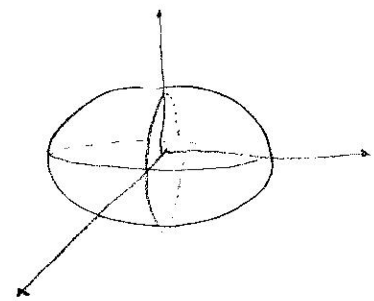
23. CONTINUOUS ON DOMAIN: $x^2 + y^2 - 4 > 0$

$$x^2 + y^2 > 4$$



25. CONTINUOUS ON DOMAIN: $-1 \le \underbrace{x^2 + y^2 + z^2}_{\text{ALWAYS NON-NEGATIVE}} \le 1$

INTERIOR + BOUNDARY OF UNIT SPHERE



27. IF $y = 0$ $\lim_{x \to 0} f(x, 0) = 0 \neq 1$

$\therefore$ CONTINUOUS EVERYWHERE EXCEPT ORIGIN.

29. $$\lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{x^2 + y^2} = \lim_{r \to 0} \frac{r^3(\cos^3\theta + \sin^3\theta)}{r^2(\cos^2\theta + \sin^2\theta)}$$

$$= \lim_{r \to 0} r(\cos^3\theta + \sin^3\theta) = \boxed{0}$$

31. $$\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2} - 1}{x^2 + y^2} = \lim_{r \to 0} \frac{e^{-r^2} - 1}{r^2} \; : \; \frac{0}{0} \text{ IND. FORM}$$

$$\xrightarrow{\text{LÔH.}} \lim_{r \to 0} \frac{-2r\, e^{-r^2}}{2r} = \boxed{-1}$$