§11.5 CHAIN RULE

ONE-VAR: $y = f(x)$, $x = g(t)$ $\Rightarrow$ $\frac{dy}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt}$

TWO-VAR: $z = f(x,y)$, $x = g(t)$, $y = h(t)$

$$\Rightarrow \boxed{\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial \cancel{g}}{\partial y} \cdot \frac{dy}{dt}}$$

WHY?

RECALL: DIFFERENTIALS $dz = \frac{\partial z}{\partial x}\, dx + \frac{\partial z}{\partial y}\, dy$

NOW DIVIDE BY dt!

(ITS GOOD NOTATION!)

BUT REALLY, WE HAVE

$$\Delta z \approx \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y$$

$$\frac{\Delta z}{\Delta t} \approx \frac{\partial f}{\partial x} \cdot \frac{\Delta x}{\Delta t} + \frac{\partial f}{\partial y} \cdot \frac{\Delta y}{\Delta t}$$

NOW TAKE $\lim_{\Delta t \to 0}$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

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