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§12.6 TRIPLE INTEGRALS IN CYLINDRICAL COORD.

3. $(-1, 1, 1) \longrightarrow (\underbrace{\sqrt{2}\cos\theta}_{=-1}, \underbrace{\sqrt{2}\sin\theta}_{=1}, 1)$

$r^2 = x^2 + y^2 = (-1)^2 + (1)^2 = 2$

$r = \sqrt{2}$

$\cos\theta = -\frac{1}{\sqrt{2}}$ $\qquad$ $\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$ $\qquad$ $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

$(r, \theta, z) = \boxed{(\sqrt{2}, \frac{3\pi}{4}, 1)}$

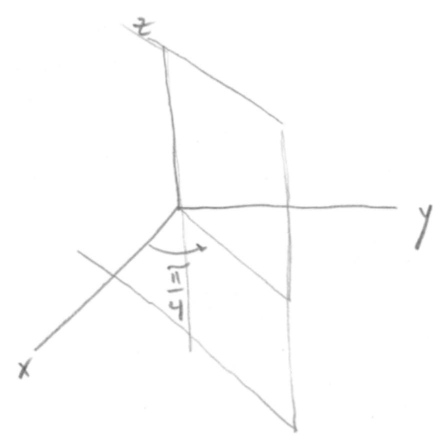
4. $(2\sqrt{3}, 2, -1) = (4\cos\theta, 4\sin\theta, -1) \longrightarrow \theta = \frac{\pi}{6}$

$r^2 = (2\sqrt{3})^2 + (2)^2 = 12 + 4 = 16$

$r = 4$

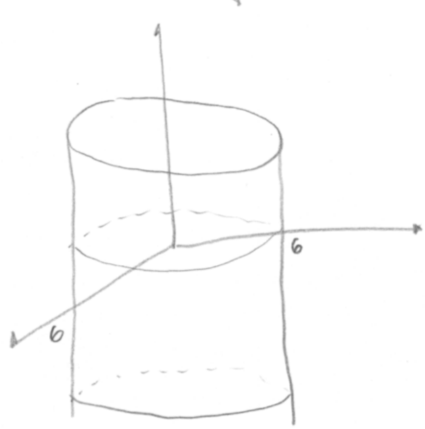
$\boxed{(4, \frac{\pi}{6}, -1)}$

5.



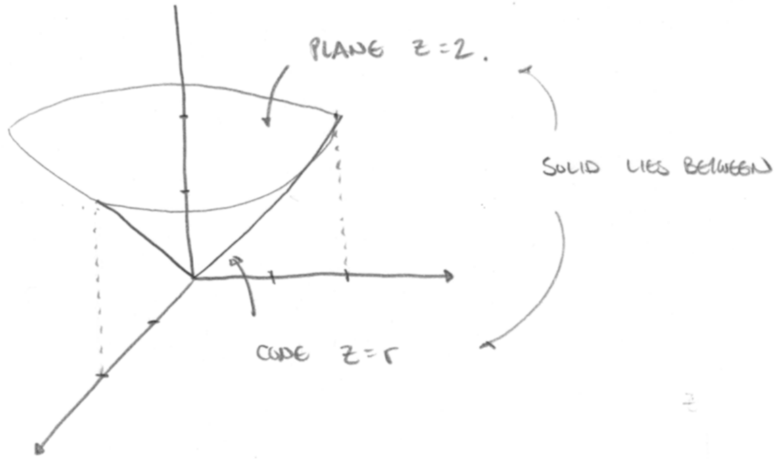
PLANE PARALLEL TO Z-AXIS, MAKES ANGLE $\frac{\pi}{4}$ W/ XZ-PLANE.

6.



CIRCULAR CYLINDER W/ RADIUS 6 $\parallel$ TO Z-AXIS.

12. $0 \le \theta \le \frac{\pi}{2}$; $r \le z \le 2$



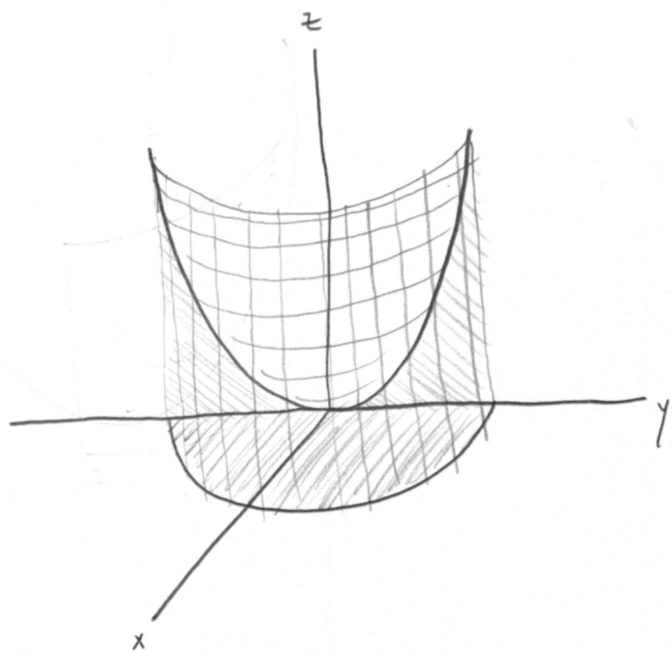
$z = r^2$ IS PARABOLOID

$z = x^2 + y^2$

15. $0 \le z \le r^2$

$0 \le r \le 2$

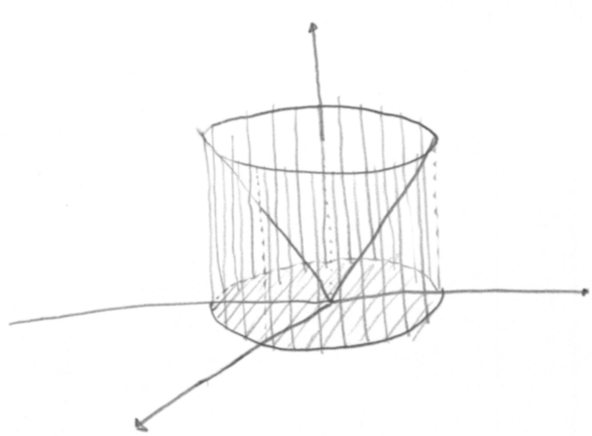
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



16. $0 \le z \le r$: $z = r$ IS CONE : $z = \sqrt{x^2+y^2}$

$0 \le \theta \le 2\pi$

$0 \le r \le 2$



$x^2\, dV = (r\cos\theta)^2\, r\, dr\, d\theta\, dz$

21.

$$\int_0^{2\pi}\int_0^{1}\int_0^{2r} r^3\cos^2\theta\, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{1} 2r^4\cos^2\theta\, dr\, d\theta = \frac{2}{5}\int_0^{2\pi}\cos^2\theta\, d\theta$$

$$= \frac{1}{5}\int_0^{2\pi} 1+\cos 2\theta\, d\theta = \boxed{\frac{2\pi}{5}}$$

22.

$$\int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta = 2\pi\int_1^{2} r\sqrt{4-r^2}\, dr$$

LET $u = 4-r^2$

$du = -2r\,dr$

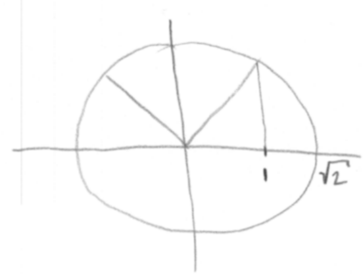
$-\frac{1}{2}du = r\,dr$

$$\rightsquigarrow \pi\int_0^{3}\sqrt{u}\, du = \frac{2\pi}{3}u^{3/2}\Big|_0^3$$

$$= \boxed{2\sqrt{3}\,\pi}$$

23.

$$\int_0^{2\pi}\int_0^{1}\int_r^{\sqrt{2-r^2}} r\, dz\, dr\, d\theta$$

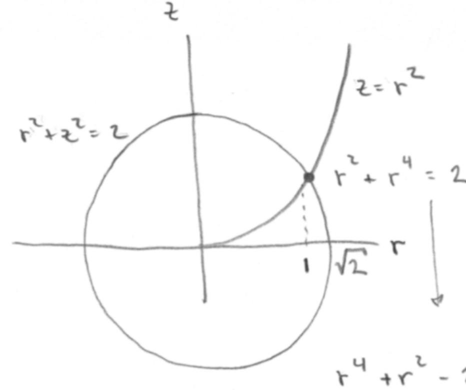


$$= 2\pi\int_0^{1} r\sqrt{2-r^2}\, dr = \pi\int_1^{2}\sqrt{u}\, du = \frac{2\pi}{3}u^{3/2}\Big|_1^2$$

$$= \boxed{\frac{2\pi}{3}\left(2\sqrt{2}-1\right)}$$

24.

$$\int_0^{2\pi}\int_0^1\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$$



$r^4+r^2-2=0$

$(r^2+2)(r^2-1)=0$

$(r^2+2)(r+1)(r-1)=0$

$r=1$

$$= 2\pi\int_0^1 r\sqrt{2-r^2} - r^3\,dr$$

$$= 2\pi\left[-\frac{1}{3}\left(2-r^2\right)^{3/2} - \frac{1}{4}r^4\right]_0^1$$

$$= 2\pi\left(-\frac{1}{3}\left(1-2\sqrt{2}\right) - \frac{1}{4}\right) = 2\pi\left(\frac{2\sqrt{2}}{3} - \frac{1}{3} - \frac{1}{4}\right)$$

$$= 2\pi\left(\frac{8\sqrt{2}-7}{12}\right) = \boxed{\frac{\pi\left(8\sqrt{2}-7\right)}{6}}$$

29.

$$\sqrt{x^2+y^2} \le z \le 2 \longrightarrow r \le z \le 2$$

$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dx\,dy \longrightarrow \int_0^{2\pi}\int_0^2 r\,dr\,d\theta$$

$$\rightsquigarrow \int_0^{2\pi}\int_0^2\int_r^2 r^2\cos\theta\, z\,dz\,dr\,d\theta = \underbrace{\int_0^{2\pi}\cos\theta\,d\theta}_{0}\int_0^2\int_r^2 r^2 z\,dz\,dr$$

$$= \boxed{0.}$$