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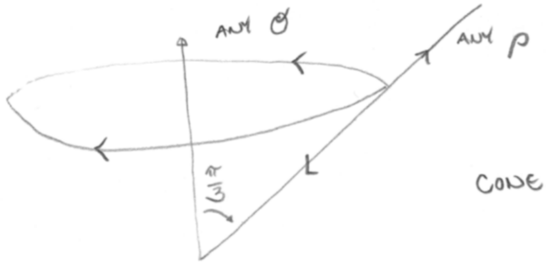
12/6/2014

5. $\phi = \frac{\pi}{3}$



CONE! (UPPER HALF CONE, REALLY)

6. SPHERE OF RADIUS 3 CENTERED AT ORIGIN.

9. (a) $z^2 = x^2 + y^2$

$$\boxed{\rho^2 \cos^2\phi = r^2}$$

(b) $$\boxed{\rho^2 \sin^2\phi \cos^2\theta + \rho^2\cos^2\phi = 9}$$

19. $$\int_0^2 \int_0^{\pi/2} \int_0^3 f(r\cos\theta, r\sin\theta, z)\, r\, dr\, d\theta\, dz$$ (CYLINDER)

20. $$\int_0^{\pi/2} \int_{\pi/2}^{2\pi} \int_1^2 f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\, \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$ (SPHERICAL)

21. $$\int_0^{\pi} \int_0^{2\pi} \int_0^5 (\rho^2)^2 \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi = \frac{5^7}{7} \cdot 2\pi \int_0^{\pi} \sin\phi\, d\phi$$

$$= \boxed{4\pi \cdot \frac{5^7}{7}}$$

22. $9 - x^2 - y^2 = 9 - r^2 = 9 - \rho^2 \sin^2\phi$

$$\int_0^{2\pi} \int_0^{\pi/2} \int_0^{3} (9 - \rho^2 \sin^2\phi)\, \rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$

$$= 2\pi \int_0^{\pi/2} 3\rho^3 \sin\phi - \frac{1}{5}\rho^5 \sin^3\phi \Big|_0^3 \, d\phi$$

$$= 162\pi \int_0^{\pi/2} \sin\phi - \frac{3}{5}\sin^3\phi \; d\phi$$

$$\sin^3\phi = (1 - \cos^2\phi)\sin\phi$$
$$= \sin\phi - \cos^2\phi \sin\phi$$

$$\sin\phi - \frac{3}{5}(\sin\phi - \cos^2\phi \sin\phi)$$

$$= 162\pi \int_0^{\pi/2} \frac{2}{5}\sin\phi - \frac{3}{5}\cos^2\phi \sin\phi \; d\phi$$

$$= 162\pi \left[-\frac{2}{5}\cos\phi + \frac{1}{5}\cos^3\phi \right]_0^{\pi/2}$$

$$= 162\pi \left(\frac{2}{5} - \frac{1}{5} \right) = \boxed{\frac{162\pi}{5}}$$

24.

$$xyz = (\rho \sin\phi \cos\theta)(\rho \sin\phi \sin\theta)(\rho\cos\phi)$$
$$= \rho^3 \sin^2\phi \cos\phi \sin\theta \cos\theta$$

$$\int_0^{2\pi}\int_0^{\pi/3}\int_2^4 \rho^5 \sin^3\phi \cos\phi \sin\theta\cos\theta \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/3} \frac{4^6-2^6}{6} \sin^3\phi\cos\phi \sin\theta\cos\theta \, d\phi\, d\theta$$

$$= \int_0^{2\pi} \frac{4^6-2^6}{6} \cdot \frac{1}{4}\sin^4\phi \Big|_0^{\pi/3} \sin\theta\cos\theta\, d\theta$$

$$= \frac{4^6-2^6}{6}\cdot\frac{1}{4}\sin^4\left(\frac{\pi}{3}\right)\cdot \underbrace{\frac{1}{2}\sin^2\theta\Big|_0^{2\pi}}_{0} = \boxed{0}$$

26. VOLUME $= \frac{4\pi}{3}a^3$. FIND AVE VALUE OF ρ:

$$\text{AVE} = \frac{3}{4\pi a^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho^3 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \frac{3}{2a^3}\int_0^{\pi} \frac{1}{4}a^4 \sin\phi\, d\phi = \frac{3}{\cancel{2}\cancel{a^3}}\cdot\frac{\cancel{a^4}}{4}\cdot\cancel{2} = \boxed{\frac{3a}{4}}$$

28.

$$\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = 2\pi \cdot \frac{8}{3} \int_{\pi/4}^{\pi/2} \sin\phi \, d\phi$$

$$= \frac{16\pi}{3}\left(\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right) = \boxed{\frac{8\sqrt{2}\,\pi}{3}}$$

37.

$$\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{\sqrt{2}} (\rho\sin\phi\cos\theta)(\rho\sin\phi\sin\theta)\rho^2\sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \frac{1}{5}\rho^5\Big|_0^{\sqrt{2}} \int_0^{\pi/2}\int_0^{\pi/4} \sin^3\phi \cos\theta \sin\theta \, d\phi \, d\theta$$

$\sin^3\phi = (1-\cos^2\phi)\sin\phi$

$$= \frac{1}{5}\rho^5\Big|_0^{\sqrt{2}} \cdot \frac{1}{2}\sin^2\theta\Big|_0^{\pi/2} \cdot \int_0^{\pi/4} \sin^3\phi \, d\phi$$

$u = \cos\phi$

$du = -\sin\phi \, d\phi$

$$= \frac{4\sqrt{2}}{5} \cdot \frac{1}{2} \int_{\frac{\sqrt{2}}{2}}^{1} 1 - u^2 \, du = \frac{2\sqrt{2}}{5}\left[u - \frac{1}{3}u^3\right]_{\frac{\sqrt{2}}{2}}^{1}$$

$$= \frac{2\sqrt{2}}{5}\left(1 - \frac{\sqrt{2}}{2} - \frac{1}{3} + \frac{2\sqrt{2}}{24}\right)$$

$$= \frac{2\sqrt{2}}{5}\left(\frac{2}{3} + \frac{2\sqrt{2} - 12\sqrt{2}}{24}\right) = \boxed{\frac{2\sqrt{2}}{5}\left(\frac{2}{3} - \frac{5\sqrt{2}}{12}\right)}$$

$\frac{-5\sqrt{2}}{12}$

$\rightarrow \frac{4\sqrt{2}}{15} - \frac{1}{3}$

12.7.4