JOHN ADAMSKI
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§ 13.6 SURFACE AREA

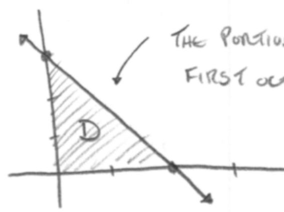
33. $3x + 2y + z = 6$ - FIRST OCTANT

$$z = 6 - 3x - 2y \qquad \frac{\partial z}{\partial x} = -3 \qquad \frac{\partial z}{\partial y} = -2$$

DOMAIN: $z = 6 - 3x - 2y = 0$

$$2y = 6 - 3y$$

$$y = -\frac{3}{2}y + 3$$



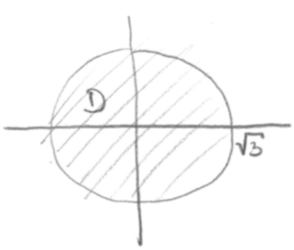
THE PORTION OF THE PLANE THAT LIES IN THE FIRST OCTANT LIES ABOVE D.

$$SA = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA = \int_0^2 \int_0^{3 - \frac{2}{2}x} \sqrt{1 + 3^2 + 2^2}\, dy\, dx = \sqrt{14} \int_0^2 3 - \frac{3}{2}x\, dx$$

$$= \sqrt{14}\left(3x - \frac{3}{4}x^2\right)\Big|_0^2 = \sqrt{14}(6 - 3) = \boxed{3\sqrt{14}}$$

35. $x + 2y + 3z = 1$ INSIDE $x^2 + y^2 = 3$

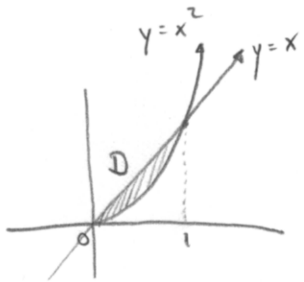
$$z = \frac{1}{3}(1 - x - 2y) \qquad \frac{\partial z}{\partial x} = -\frac{1}{3}, \quad \frac{\partial z}{\partial y} = -\frac{2}{3}$$



$$SA = \iint_D \sqrt{1 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}\, dA = \frac{\sqrt{14}}{3} \int_0^{2\pi} \int_0^{\sqrt{3}} r\, dr\, d\theta$$

$$= \frac{\sqrt{14}}{3} \cdot 2\pi \cdot \frac{r^2}{2}\Big|_0^{\sqrt{3}} = \boxed{\sqrt{14}\pi}$$

36.



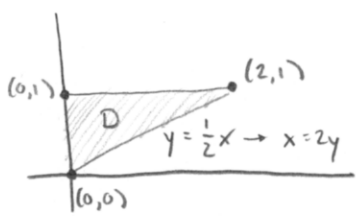
$z = \sqrt{x^2+y^2}$

$\dfrac{\partial z}{\partial x} = \dfrac{x}{\sqrt{x^2+y^2}}, \quad \dfrac{\partial z}{\partial y} = \dfrac{y}{\sqrt{x^2+y^2}}$

$SA = \iint_D \sqrt{1 + \dfrac{x^2+y^2}{x^2+y^2}}\, dA = \sqrt{2}\iint_D dA = \sqrt{2}\int_0^1 x - x^2\, dx = \sqrt{2}\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1$

$= \boxed{\dfrac{\sqrt{2}}{6}}$

38.



$z = 1 + 3x + 2y^2$

$\dfrac{\partial z}{\partial x} = 3 \qquad \dfrac{\partial z}{\partial y} = 4y$

$SA = \iint_D \sqrt{1 + 3^2 + (4y)^2}\, dA = \iint_D \sqrt{10+16y^2}\, dA = \int_0^1\int_0^{2y} \sqrt{10+16y^2}\, dx\, dy$

$= \int_0^1 2y\sqrt{10+16y^2}\, dy$ let $u = 10 + 16y^2$, $du = 32y$ $\longrightarrow \int_{10}^{26} \frac{1}{16}\sqrt{u}\, du$

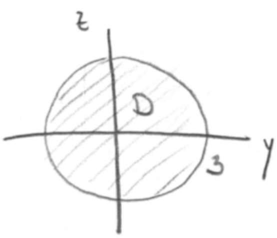
$= \frac{1}{16}\cdot\frac{2}{3}u^{3/2}\Big|_{10}^{26} = \boxed{\frac{1}{24}\left(26^{3/2} - 10^{3/2}\right)}$

39. $z = xy$ WITHIN UNIT CIRCLE

$$SA = \iint_D \sqrt{1 + x^2 + y^2}\, dA = \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\; r\, dr\, d\theta$$

$$= 2\pi \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} \left(1 + r^2\right)^{3/2} \Big|_0^1 = \boxed{\frac{2\pi}{3}\left(2^{3/2} - 1\right)}$$

40.



$$SA = \iint_D \sqrt{1 + \left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2}\; dA$$

$$= \iint_D \sqrt{1 + 4y^2 + 4z^2}\; dA = \int_0^{2\pi}\int_0^3 \sqrt{1+4r^2}\; r\, dr\, d\theta$$

$u = 1 + 4r^2$

$du = 8r\,dr$

$$\to \frac{\pi}{4}\int_1^{37} \sqrt{u}\, dr = \frac{\pi}{4} \cdot \frac{2}{3} u^{3/2} \Big|_1^{37} = \boxed{\frac{\pi}{6}\left(37^{3/2} - 1\right)}$$