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§13.6 PARAMETRIC SURFACES (PART I)

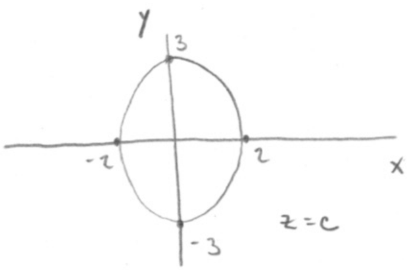
1. $\vec{r}(u,v) = (u+v)\hat{\imath} + (3-v)\hat{\jmath} + (1+4u+5v)\hat{k}$

$= \langle 0, 3, 1 \rangle + u\langle 1, 0, 4 \rangle + v\langle 1, -1, 5 \rangle$

$\boxed{\text{THIS IS THE PLANE THROUGH } (0,3,1) \text{ PARALLEL TO BOTH } \langle 1,0,4 \rangle \text{ AND } \langle 1,-1,5 \rangle.}$

2. $\vec{r}(u,v) = 2\sin u\,\hat{\imath} + 3\cos u\,\hat{\jmath} + v\hat{k}, \quad 0 \le v \le 2.$

SETTING $v = c$ CONSTANT, WE GET A SPACE CURVE $\vec{r}(u) = \langle 2\sin u, 3\cos u, c \rangle$ WHICH IS AN ELLIPSE IN THE PLANE $z = c$.

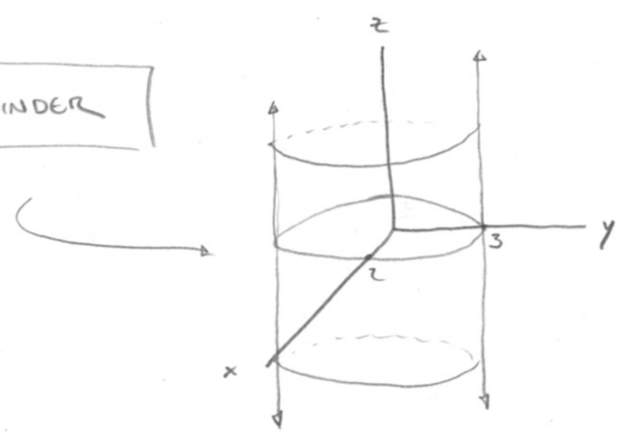


SETTING $u = c$ CONSTANT, WE GET SPACE CURVE

$\vec{r}(v) = \langle \underbrace{2\sin(c)}_{\text{CONSTANT}}, \underbrace{3\cos(c)}_{\text{CONSTANT}}, v \rangle$

WHICH IS A VERTICAL LINE ($\parallel$ TO z-AXIS)

$\therefore$ GRAPH IS $\boxed{\text{ELLIPTIC RIGHT CYLINDER}}$



11. $\boxed{\text{IV}}$: GRID CURVES $u = u_0$ ARE HELIXES (CORKSCREWS) CIRCLING THE z-AXIS AS THEY TRAVEL UPWARDS

$v = v_0$ ARE LINES THROUGH z-AXIS $\parallel$ TO xy-PLANE

12. $\boxed{\text{I}}$: GRID CURVES $u = u_0$ ARE CIRCLES AROUND THE Z-AXIS WITH RADIUS u_0

$v = v_0$ IS SIN CURVE THROUGH ORIGIN IN PLANE CONTAING Z-AXIS AND VECTOR $\langle \cos v_0 , \sin v_0 , 0 \rangle$

15. $\vec{r}(u,v) = u(\hat{i} - \hat{j}) + v(\hat{j} - \hat{k})$

$= u\langle 1, -1, 0\rangle + v\langle 0, 1, -1\rangle$

$$\boxed{\vec{r}(u,v) = \langle u , v-u , -v \rangle}$$

16. $\vec{r}(u,v) = \langle 0,-1,5\rangle + u\langle 2,1,4\rangle + v\langle -3,2,5\rangle$

$$\boxed{\vec{r}(u,v) = \langle 2u-3v , -1+u+2v , 5+4u+5v \rangle}$$

17. $4x^2 - 4y^2 - z^2 = 4$

$x^2 = y^2 + \frac{z^2}{4} + 1$, $x > 0$ (GIVEN)

$x = \sqrt{y^2 + \frac{z^2}{4} + 1}$

LET $y = u$, $z = v$, $x = \sqrt{u^2 + \frac{v^2}{4} + 1}$

$$\boxed{\vec{r}(u,v) = \left\langle \sqrt{u^2 + \frac{v^2}{4} + 1} , u , v \right\rangle}$$

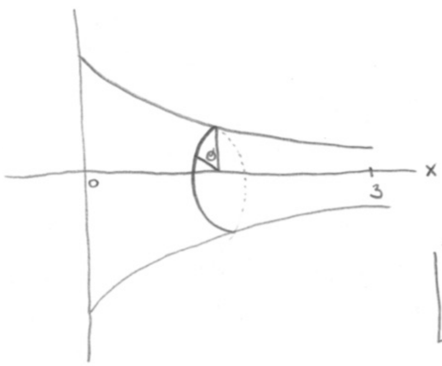
18. $x^2 + 2y^2 + 3z^2 = 1$

$$y^2 = 1 - \frac{x^2}{2} - \frac{3z^2}{2}, \quad y \le 0 \quad \text{(GIVEN)}$$

$$y = -\sqrt{1 - \frac{x^2}{2} - \frac{3z^2}{2}} \qquad \text{LET } x = u, \ z = v, \ y = \ldots$$

$$\boxed{\vec{r}(u,v) = \left\langle u, -\sqrt{1 - \frac{u^2}{2} - \frac{3v^2}{2}}, v \right\rangle}$$

25.



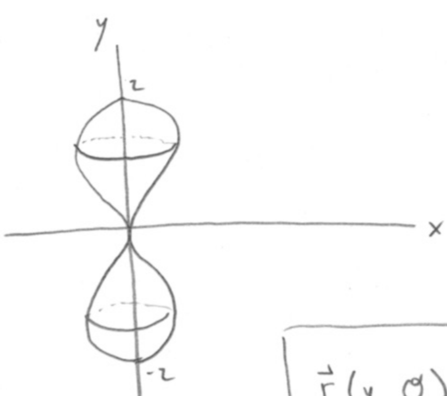
LET $x = x$

$y = e^{-x}\cos\theta$

$z = e^{-x}\sin\theta$

$$\boxed{\vec{r}(x,\theta) = \langle x, e^{-x}\cos\theta, e^{-x}\sin\theta \rangle, \quad 0 \le x \le 3}$$

26.



LET $y = y$

$x = (4y^2 - y^4)\cos\theta$

$z = (4y^2 - y^4)\sin\theta$

$$\boxed{\vec{r}(y,\theta) = \langle (4y^2 - y^4)\cos\theta, y, (4y^2 - y^4)\sin\theta \rangle, \quad -2 \le y \le 2}$$