§ 8.1 SEQUENCES

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12/13/14

10. $\lim_{n\to\infty} \frac{n^3}{n^3+1} = \lim_{n\to\infty} \frac{n^3+1}{n^3+1} - \frac{1}{n^3+1} = 1 - 0 = \boxed{1}$

11. $\lim_{n\to\infty} \frac{3+5n^2}{n+n^2} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(\frac{3}{n^2}+5\right)}{\cancel{n^2}\left(\frac{1}{n}+1\right)} = \boxed{5}$

12. $\lim_{n\to\infty} \frac{n^3}{n+1} = \lim_{n\to\infty} \frac{\cancel{n}(n^2)}{\cancel{n}(1+\frac{1}{n})} = \boxed{\infty \text{ (DIVERGES)}}$

13. $\lim_{n\to\infty} \tan\left(\frac{2n\pi}{1+8n}\right) = \tan\left(\lim_{n\to\infty} \frac{2n\pi}{1+8n}\right) = \tan\left(\frac{\pi}{4}\right) = \boxed{1}$

20. $\lim_{n\to\infty} \cos\left(\frac{2}{n}\right) = \cos\left(\lim_{n\to\infty} \frac{2}{n}\right) = \cos(0) = \boxed{1}$

24. $\lim_{n\to\infty} \ln(n+1) - \ln(n) = \lim_{n\to\infty} \ln\left(\frac{n+1}{n}\right) = \ln\left(\lim_{n\to\infty} \frac{n+1}{n}\right)$

$= \ln(1) = \boxed{0}$

26. $$\frac{-1}{2^n} \le \frac{\cos(n\pi)}{2^n} \le \frac{1}{2^n}$$

$$0 = \lim_{n\to\infty} \frac{-1}{2^n} \le \lim_{n\to\infty} \frac{\cos(n\pi)}{2^n} \le \lim_{n\to\infty} \frac{1}{2^n} = 0$$

$$\downarrow$$

$$\boxed{0}$$

27. CONSIDER $f(x) = \left(1 + \frac{2}{x}\right)^x$. $f(n) = a_n$.

$$\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x : 1^\infty \text{ IND. FORM}$$

$$= e^{\lim_{x \to \infty} x \operatorname{Log}\left(1 + \frac{2}{x}\right)} = e^{\lim_{x \to \infty} \frac{\operatorname{Log}\left(1 + \frac{2}{x}\right)}{1/x}} : \frac{0}{0}$$

$$\xrightarrow{\text{L'HÔ}} = e^{\lim_{x \to \infty} \frac{\frac{1}{1 + \frac{2}{x}} \cdot \left(\cancel{-}\frac{2}{\cancel{x^2}}\right)}{\cancel{-}\frac{1}{\cancel{x^2}}}} = e^{\lim_{x \to \infty} \frac{2x}{x + 2}} = \boxed{e^2}$$