Name: *ANSWER KEY*

Math 203 Calculus III

10/2/2014

Quiz 1

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. (4 points) Find the angle between the vectors $\vec{a} = \langle 4, 0, 2 \rangle$ and $\vec{b} = \langle 2, -1, 0 \rangle$. You may leave your answer as a trigonometric expression.

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\vec{a} \cdot \vec{b} = \langle 4, 0, 2 \rangle \cdot \langle 2, -1, 0 \rangle = (4)(2) + (0)(-1) + (2)(0) = 8$$

$$|\vec{a}| = \sqrt{4^2 + 0^2 + 2^2} = \sqrt{20}$$

$$|\vec{b}| = \sqrt{2^2 + (-1)^2 + 0^2} = \sqrt{5}$$

$$\Rightarrow \quad 8 = \sqrt{20}\sqrt{5} \cos\theta = \sqrt{100} \cos\theta = 10 \cos\theta$$

$$\frac{8}{10} = \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{4}{5}\right)}$$

2. (4 points) Give parametric equations for the line that passes through the points $(2, 1, 2)$ and $(7, -1, 8)$.

DIRECTION OF LINE : $\langle 7, -1, 8 \rangle - \langle 2, 1, 2 \rangle = \langle 5, -2, 6 \rangle$

LINE IN VECTOR FORM : $\vec{r}(t) = \langle 2, 1, 2 \rangle + t \langle 5, -2, 6 \rangle$

$$= \langle \underbrace{2 + 5t}_{x}, \underbrace{1 - 2t}_{y}, \underbrace{2 + 6t}_{z} \rangle$$

$$\boxed{x = 2 + 5t, \quad y = 1 - 2t, \quad z = 2 + 6t}$$

3. Consider the three points $P(-2,3,2)$, $Q(-1,1,5)$, and $R(0,4,2)$.

(a) (4 points) Give an equation for the plane through P, Q, and R.

NORMAL VECTOR TO PLANE IS NORMAL TO $\overrightarrow{PQ}$ & $\overrightarrow{PR}$

SO USE $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle -1+2, 1-3, 5-2 \rangle \times \langle 0+2, 4-3, 2-2 \rangle$

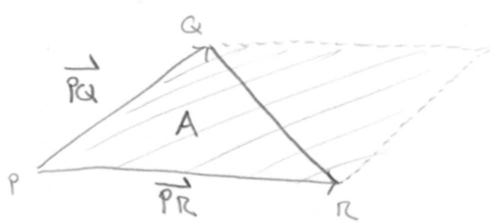
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & 0 \end{vmatrix} = \langle -3, 6, 5 \rangle$$

$\therefore \langle -3, 6, 5 \rangle \cdot \langle x+2, y-3, z-2 \rangle = 0$

$-3(x+2) + 6(y-3) + 5(z-2) = 0$

$$\boxed{-3x + 6y + 5z = 34}$$

(b) (4 points) Find the area of the triangle with vertices P, Q, and R.



$A = \frac{1}{2}$ (AREA OF PARALLELOGRAM DETERMINED BY $\overrightarrow{PQ}$ & $\overrightarrow{PR}$)

$A = \frac{1}{2} \left| \overrightarrow{PQ} \times \overrightarrow{PR} \right| = \frac{1}{2} \left| \langle -3, 6, 5 \rangle \right|$

$= \frac{1}{2}\sqrt{(-3)^2 + 6^2 + 5^2} = \boxed{\frac{1}{2}\sqrt{70}}$

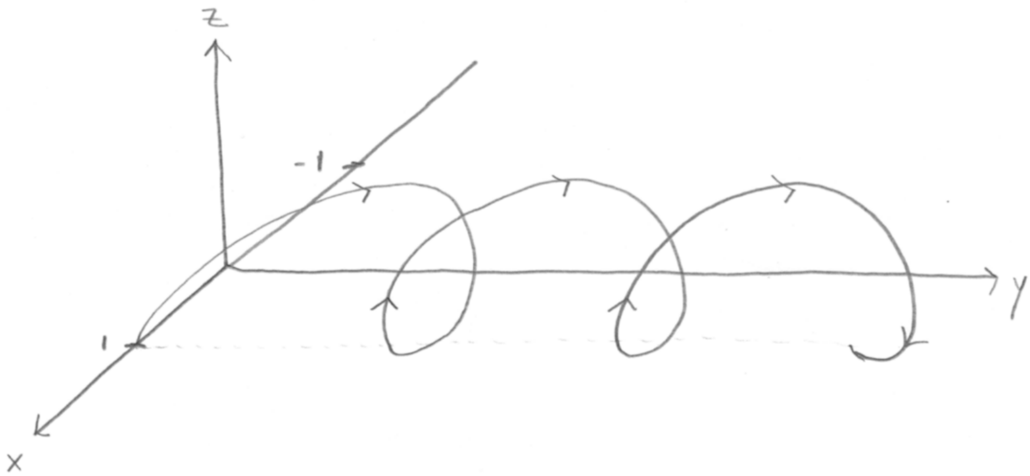
4. Consider the parametrized curve

$$x(t) = \cos t, \quad y(t) = t, \quad z(t) = \sin t, \quad 0 \le t \le 6\pi, \qquad \text{(parametric)}$$

i.e.,

$$\vec{r}(t) = \langle \cos t, t, \sin t \rangle, \quad 0 \le t \le 6\pi. \qquad \text{(vector)}$$

(a) (4 points) Sketch the curve.



(b) (4 points) Find the unit tangent vector to the curve when $t = 2\pi$.

$\vec{r}\,'(t) = \langle -\sin t, 1, \cos t \rangle$

$\vec{r}\,'(2\pi) = \langle -\sin(2\pi), 1, \cos(2\pi) \rangle$

$= \langle 0, 1, 1 \rangle$

BUT $|\langle 0, 1, 1 \rangle| = \sqrt{2}$

SO TO MAKE IT <u>UNIT</u> VECTOR : $\boxed{\frac{1}{\sqrt{2}} \langle 0, 1, 1 \rangle}$