Name: * ANSWER KEY * 10/30/2014

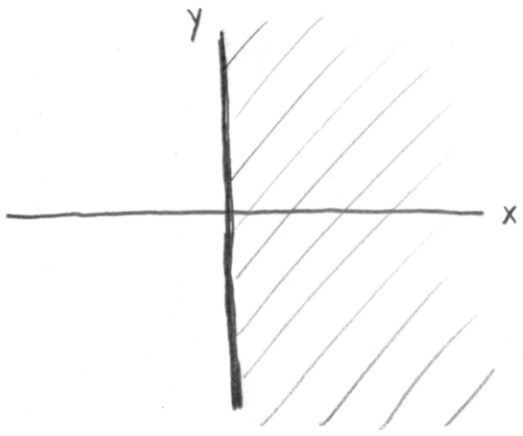
Math 203 Calculus III Quiz 2

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Let $f(x,y) = y - \sqrt{x}$.

   (a) (4 points) Sketch the domain of f.

$f(x,y)$ IS DEFINED FOR ALL (x,y) SUCH THAT $x \geq 0$

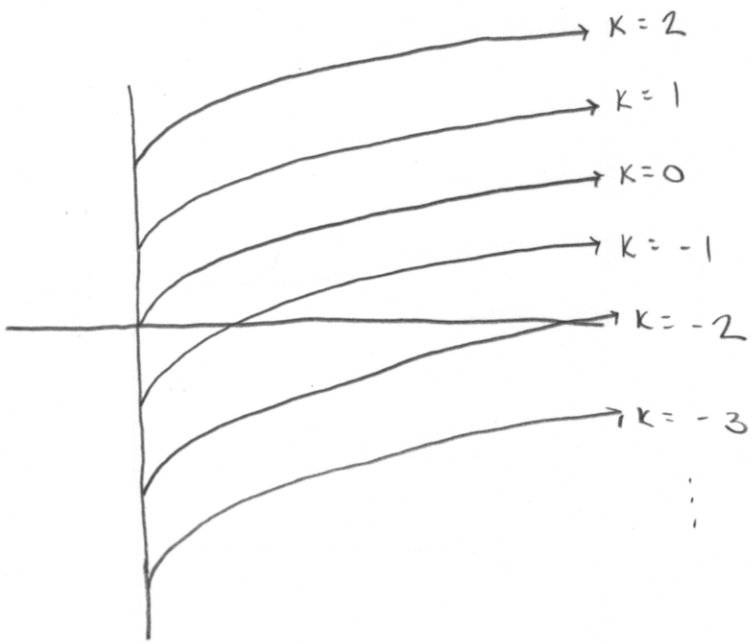


   (b) (4 points) Draw a contour map of the function showing several level curves.

$f(x,y) = K$

$y - \sqrt{x} = K$

$\underline{\underline{y = \sqrt{x} + K}}$



NOTE THAT THE LEVEL CURVES FILL THE ENTIRE DOMAIN.

2. (4 points) Show that the following function is not continuous at (0,0).

$$f(x,y) = \begin{cases} \dfrac{x^4 - 2y^2}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

THE FUNCTION IS ONLY CONTINUOUS AT $(0,0)$ IF

$$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0).$$

SINCE $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{y\to 0} \dfrac{-2y^2}{y^2} = -2$

ALONG $x = 0$

DIFFERENT!

AND $f(0,0) = 0$, THE FUNCTION IS NOT CONTINUOUS AT $(0,0)$.

(IN FACT, THE LIMIT DOES NOT EVEN EXIST SINCE APPROACHING $(0,0)$ ALONG DIFFERENT LINES & CURVES PRODUCES DIFFERENT LIMIT VALUES.)

3. (4 points) Give an equation for the tangent plane to the surface $z = \dfrac{2x+3}{4y+1}$ at the point $(0,0,3)$.

(x_0, y_0, z_0)

EQUATION OF TANGENT PLANE:

$$z = z_0 + f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$$

WE HAVE $z = \dfrac{2}{4y+1}\,x + \dfrac{3}{4y+1} = (2x+3)(4y+1)^{-1}$

AND $f_x = \dfrac{2}{4y+1}$, $f_y = \dfrac{-4(2x+3)}{(4y+1)^2}$

$f_x(0,0) = 2$ $\quad f_y(0,0) = -12$

$\therefore$ $\boxed{z = 3 + 2x - 12y}$

4. Let $z = \ln \sqrt{x^2 + y^2}$

(a) (4 points) Find the differential dz.

$$dz = f_x\,dx + f_y\,dy$$

$$f_x = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{x^2+y^2}$$

$$f_y = \frac{y}{x^2+y^2} \quad \text{SIMILARLY.}$$

$$\therefore \boxed{dz = \frac{x}{x^2+y^2}\,dx + \frac{y}{x^2+y^2}\,dy}$$

(b) (4 points) Use dz to approximate the change in z as (x, y) changes from $(1, 3)$ to $(0.9, 3.1)$.

$$x = 1, \quad y = 3, \quad dx = 0.9 - 1 = -0.1,$$
$$dy = 3.1 - 3 = 0.1.$$

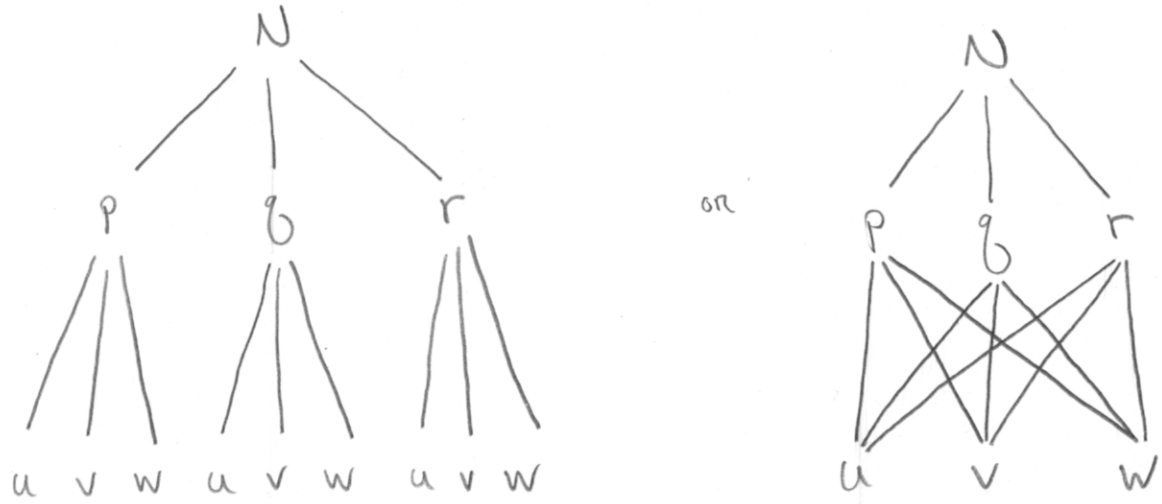
$$dz = \frac{1}{1^2+3^2}(-0.1) + \frac{3}{1^2+3^2}(0.1)$$

$$= \frac{-0.1}{10} + \frac{0.3}{10} = \boxed{\frac{0.2}{10} \text{ OR } 0.02}$$

5. Suppose

$$N = pq + q^r, \quad p = u + vw, \quad q = v + uw, \quad r = w + uv.$$

(a) (4 points) Draw a tree diagram (or a "bush diagram") to show how N is a function of u, v, and w.



(b) (4 points) Find $\frac{\partial N}{\partial w}$ in terms of p, q, r, u, v, and w.

$$\frac{\partial N}{\partial w} = \frac{\partial N}{\partial p} \cdot \frac{\partial p}{\partial w} + \frac{\partial N}{\partial q} \cdot \frac{\partial q}{\partial w} + \frac{\partial N}{\partial r} \cdot \frac{\partial r}{\partial w}$$

$$\frac{\partial N}{\partial w} = (q)(v) + (p + rq^{r-1})(u) + (q^r \ln q)(1)$$

$$\boxed{\frac{\partial N}{\partial w} = qv + (p + rq^{r-1})u + q^r \ln q}$$