Name:____ **10/12/2017**

Math 203 Calculus III **Exam 1**

1. (8 points) Find equations of the spheres with center $(1, -2, 3)$ that touch

   (a) the plane $x = -1$;

   (b) the origin.

2. (6 points) Let $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ be vectors. State whether each of the following expressions is meaningful or not. If yes, state whether the result is a scalar or a vector.

   (a) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

   (b) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

   (c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

   (d) $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$

   (e) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$

   (f) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

3. (8 points) Consider the two planes P_1 and P_2.

$$\begin{aligned} P_1 &: \quad 2x - 4y + 5z = 2 \\ P_2 &: \quad 5x + 2y - 4z = 5 \end{aligned}$$

(a) Find the angle at which the two planes intersect.

(b) Give either a vector equation or parametric equations for the line of intersection of P_1 and P_2.
Hint: It is easy to see that one point on the line of intersection is $(1, 0, 0)$.

4. (4 points) Give a vector equation and parametric equations for the line *segment* from $(9,-2,5)$ to $(7,3,-6)$.

5. (8 points) Suppose a line l is parametrized by the equations

$$x = 4 + t, \qquad y = 3 - 2t, \qquad z = t - 7,$$

and a plane P is described by the equation

$$2x - 6y + z = 10.$$

Give parametric equations for the line through the point $(5,8,5)$ that is parallel to the plane P and perpendicular to the line l.

6. (8 points) Give an equation for the plane that contains the three points $(2, 1, 2)$, $(9, 1, 7)$ and $(6, 4, 6)$.

7. (4 points) Sketch the domain of the function $f(x, y) = \arcsin(x^2 + y^2 - 3)$.

8. (10 points) Let C be the space curve described by the vector-valued function

$$\mathbf{r}(t) = \langle \cos t, 3\sin t, t \rangle, \qquad 0 \leq t \leq 6\pi$$

(a) Give parametric equations for the tangent line to C at the point $(0, 3, 5\pi/2)$.

(b) Sketch the curve C and the tangent line from part (a) below.

9. (8 points) (a) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + xy + y^2}$$

(b) Show that the following limit does exist, and find the limit.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2 + y^2}$$

10. (6 points) Match each of the following equations with its graph (labeled A-F) and its contour map (labeled I-VI).

(a) $z = \sin(xy)$

(b) $z = e^x \cos y$

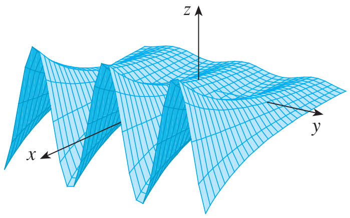
(c) $z = \sin(x - y)$

(d) $z = \sin(x) - \sin(y)$

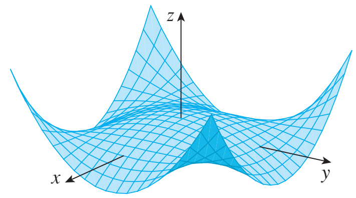
(e) $z = (1 - x^2)(1 - y^2)$

(f) $z = \dfrac{x - y}{1 + x^2 + y^2}$

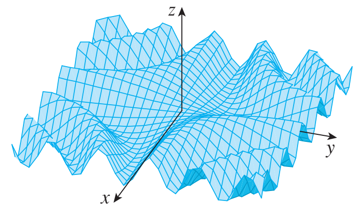
A



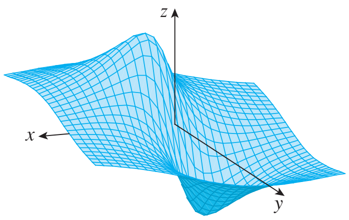
B



C



D



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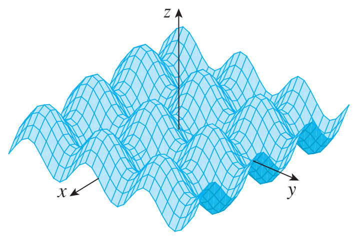

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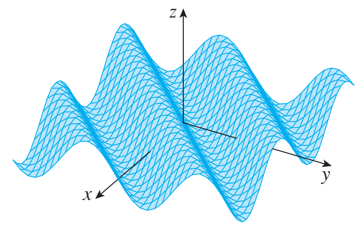
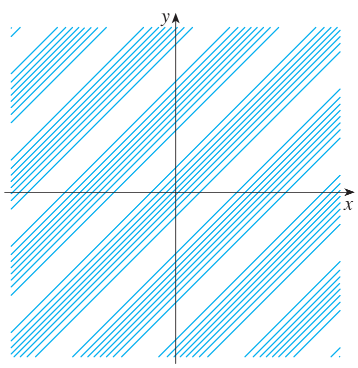


I



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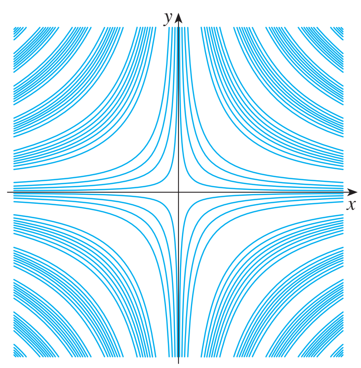
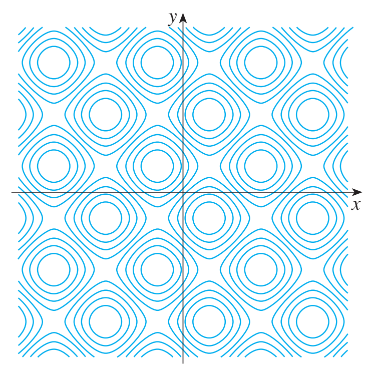
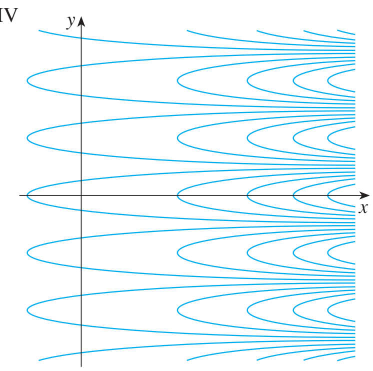


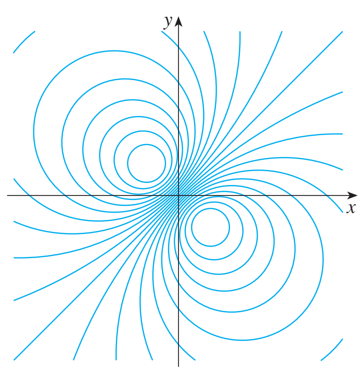
III



IV



V



VI