Name:____ **12/19/2017**
Math 203 Calculus III **Final Exam**

Part I: Answer all 7 questions (10 points each)

1. (a) Give an equation for the plane through the following three points.

$$A = (1,1,1) \quad B = (-3,2,5) \quad C = (4,-1,6)$$

(b) The line l has the following parametric equations.

$$x = 4 - 2t, \quad y = 3 - 5t, \quad z = -3 - 3t \qquad (l)$$

Determine whether each of the following planes is parallel to l, perpendicular to l, or neither .

i. $x + 5y - 4z = -9$
ii. $4x + 10y + 6z = 16$
iii. $-25x - 11y + 35z = 23$

(c) Find parametric equations for the line through $(2,4,6)$ that is perpendicular to the plane

$$x - y + 3z = 7.$$

2. (a) Give a unit tangent vector to the following parametric curve at the point $(4,2,7)$.

$$\mathbf{r}(t) = \langle t^3 + 3t, t^2 + 1, 3t + 4 \rangle$$

(b) Give an equation for the tangent plane to the following surface at the point $(0,0,3)$.

$$z = \frac{2x+3}{4y+1}$$

3. For each of the following, evaluate the limit or show that it does not exist.

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{5y^4 \cos^2 x}{x^4 + y^4}$

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

4. Suppose that over a certain region of space the electrical potential V is given by

$$V(x,y,z) = 5x^2 - 3xy + xyz.$$

(a) Find the rate of change of V at the point $P = (3,4,5)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

(b) In which direction does V change most rapidly at P?

(c) Now suppose that

$$x = s + t, \quad y = 2s - 3t, \quad z = st^2.$$

Find $\dfrac{\partial V}{\partial t}$ at the point $(s,t) = (1,1)$.

5. Find all critical points for the function

$$f(x,y) = y^2 - 2y\cos x$$

and classify each one as a local maximum, local minimum, or saddle point.

6. (a) Find the volume of the solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$.

(b) Find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 4$.

7. For each of the following series, state whether it is absolutely convergent, conditionally convergent, or divergent. Name a test which supports each conclusion, and show the work to apply the test.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{5n+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

Part II: Answer 3 out of 5 questions (10 points each)

8. Find the area of the following parametric surface.

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, v \rangle, \quad u^2 + v^2 \leq 1$$

9. (a) In spherical coordinates, the cone $9z^2 = x^2 + y^2$ has equation $\phi = c$. Find c.

(b) Find

$$\iint_R (x^2 + y^2 + z^2)^{3/2} \, dV,$$

where R is the region inside the sphere $x^2 + y^2 + z^2 = 3$ and inside the cone $z = \dfrac{\sqrt{x^2 + y^2}}{3}$.

10. Let

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

(a) Show that $f_x(0, 0)$ and $f_y(0, 0)$ both exist. Note that you must use the definition of partial derivatives as limits.

(b) Explain why f is not differentiable at $(0, 0)$.

11. (a) Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

(b) Find the mass of the lamina that occupies the triangular region enclosed by the following lines and has density $\delta(x, y) = x^2$.

$$x = 0, \quad y = x, \quad 2x + y = 6$$

12. For each of the following functions, find the maximum and minimum values of the function on the circular disk $x^2 + y^2 \leq 1$. Do this by first sketching the level curves and gradients.

(a) $f(x, y) = x + y + 1$

(b) $f(x, y) = x^2 + 2y^2$