Name:____ **10/8/2017**

Math 203 Calculus III **Practice Exam 1**

1. Find equations of the spheres with center $(2, 3, 6)$ that touch

   (a) the xy-plane;

   (b) the yz-plane;

   (c) the xz-plane.

2. (a) Find the acute angle between the two vectors $\mathbf{a} = \langle 4, 0, 2\rangle$ and $\mathbf{b} = \langle 2, -1, 0\rangle$.

   (b) Find the acute angle between the following lines.

$$2x - y = 3 \qquad 3x + y = 7$$

3. Find the scalar and vector projections of $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ onto $\mathbf{a} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$, i.e. $\text{comp}_{\mathbf{a}}\,\mathbf{b}$ and $\text{proj}_{\mathbf{a}}\,\mathbf{b}$.

4. Find two unit vectors orthogonal to both $\mathbf{u} = \langle 3, 2, 1\rangle$ and $\mathbf{v} = \langle 1, 1, 0\rangle$.

5. Find the volume of the parallelepiped ("parallelo-box") determined by the vectors $\mathbf{a} = \langle 1, 2, 3\rangle$, $\mathbf{b} = \langle 1, 1, 2\rangle$, $\mathbf{c} = \langle 2, 1, 4\rangle$.

6. Use the scalar triple product to determine whether the points $A(1,3,2,$ $B(3,-1,6)$, $C(5,2,0)$, and $D(3,6,-4)$ lie in the same plane.

7. Find a vector equation and parametric equations for the line through the point $(3,-4,6)$ and

   (a) perpendicular to the plane $x+3y-z=11$.

   (b) parallel to the vector $\mathbf{v} = \langle 1,3,-1 \rangle$.

   (c) Briefly explain why the same answer works for both parts (a) and (b).

8. Find a vector equation and parametric equations for the line *segment* from $(7,3,-1)$ to $(-2,4,5)$.

9. Find parametric equations for the line through the point $(0, 1, 2)$ that is parallel to the plane $x+y+z = 2$ and perpendicular to the line

$$x = 1 + t, \qquad y = 1 - t, \qquad z = 2t.$$

10. Find parametric equations for the tangent line to the curve $\mathbf{r}(t) = \langle t^3 + 3t, t^2 + 1, 3t + 4\rangle$ at the point $(4, 2, 7)$.

11. Sketch the domain of the function $f(x, y) = \dfrac{\sqrt{y - x^2}}{1 - x^2}$.

12. (a) Show that the following limit does not exist.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$$

(b) Show that the following limit does exist, and find the limit.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$$