Name:____ **11/16/2017**

Math 203 Calculus III **Practice Exam 2**

1. Let $f(x, y) = 1 + x\ln(xy - 5)$. Explain why f is differentiable at the point $(2, 3)$. Then find the linearization $L(x, y)$ of the function at that point.

2. Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

3. Let

$$P = \sqrt{u^2 + v^2 + w^2}, \quad u = xe^y, \quad v = ye^x, \quad w = e^{xy}.$$

Use the Chain Rule for several variables to find $\dfrac{\partial P}{\partial x}$ and $\dfrac{\partial P}{\partial y}$ when $x = 0$ and $y = 2$.

4. Find the maximum rate of change of $f(x, y, z) = \tan^{-1}(xyz)$ at the point $(1, 2, 1)$ and the direction in which it occurs.

5. After drifting, the height h in inches of the snow at point (x, y) in a parking lot is

$$h(x, y) = 4 + x^2 - \ln(y^2 + 1).$$

(a) Find the rate at which the height of the snow at the point $(3, 1)$ changes per unit distance in the direction toward the point $(4, 0)$.

(b) Suppose a person is walking in the parking lot and their position at time t is given by the following parametric equations.

$$x = 2t, \quad y = \sin t, \quad 0 \leq t$$

Find the rate at which the height of the snow the person is walking through is changing per unit time when the person is at the point $(\pi, 1)$.

6. Find the absolute maximum and absolute minimum value of

$$f(x, y) = x^4 + y^4 - 4xy + 2, \quad 0 \le x \le 3, \quad 0 \le y \le 2.$$

7. Find all local maxima, local minima, and saddle points for

$$f(x, y) = 2x^4 - x^2 + 3y^2.$$

8. Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.

9. Find the volume of the solid that lies below the surface $z = x^2 y$ and above the triangular region in the xy-plane with vertices $(0, 0)$, $(4, 2)$ and $(1, 5)$.

10. Evaluate the following double integral over the region D.

$$\iint_D \frac{x^2 \sin(x^2 + y^2)}{x^2 + y^2}\, dA, \quad D = \{4 \leq x^2 + y^2 \leq 9, y \geq 0\}$$