

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Give equations for the spheres with center  $(2, -5, 3)$  that touch

(a) (4 points) the  $xz$ -plane.

(b) (4 points) the origin.

2. (6 points) Find the angle between the vectors  $\vec{a} = \langle \sqrt{3}, 1 \rangle$  and  $\vec{b} = \langle 1, \sqrt{3} \rangle$ .

3. (6 points) Let  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ , and  $\mathbf{d}$  be vectors. State whether each of the following expressions is meaningful or not. If yes, state whether the result is a scalar or a vector.

(a)  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

(b)  $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

(c)  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

(d)  $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$

(e)  $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$

(f)  $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

4. (6 points) Find a *unit vector* that is orthogonal to the vector  $\vec{v} = \langle 1, 2, 3 \rangle$ .

5. Consider the three points  $P(1, 0, 1)$ ,  $Q(-2, 1, 3)$ , and  $R(4, 2, 5)$ .
- (a) (6 points) Find a vector orthogonal to the plane containing  $P$ ,  $Q$ , and  $R$ .

- (b) (4 points) Find the area of the triangle with vertices  $P$ ,  $Q$ , and  $R$ .