Name: * ANSWER KEY * 9/13/2018

Math 20300-ST Calculus III **Quiz 1**

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Give equations for the spheres with center $(2, -5, 3)$ that touch
   (a) (4 points) the xz-plane.

DISTANCE FROM $(2,-5,3)$ TO XZ-PLANE : $|y| = |-5| = 5$

SPHERE WITH CENTER $(2,-5,3)$ & RADIUS 5 :

$$\boxed{(x-2)^2 + (y+5)^2 + (z-3)^2 = 25}$$

   (b) (4 points) the origin.

DISTANCE FROM $(2,-5,3)$ TO ORIGIN : $\sqrt{(2-0)^2 + (-5-0)^2 + (3-0)^2}$

$$= \sqrt{4+25+9} = \sqrt{38}$$

$$\boxed{(x-2)^2 + (y+5)^2 + (z-3)^2 = 38}$$

2. (6 points) Find the angle between the vectors $\vec{a} = \langle\sqrt{3}, 1\rangle$ and $\vec{b} = \langle 1, \sqrt{3}\rangle$.

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\langle\sqrt{3}, 1\rangle\cdot\langle 1, \sqrt{3}\rangle = \sqrt{3+1}\,\sqrt{1+3}\,\cos\theta$$

$$2\sqrt{3} = 4\cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{6}}$$

3. (6 points) Let **a**, **b**, **c**, and **d** be vectors. State whether each of the following expressions is meaningful or not. If yes, state whether the result is a scalar or a vector.

(a) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

SCALAR

(b) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

NO

(c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

VECTOR

(d) $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$

NO

(e) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$

NO

(f) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

SCALAR

4. (6 points) Find a *unit vector* that is orthogonal to the vector $\vec{v} = \langle 1, 2, 3 \rangle$.

SUPPOSE $\langle x, y, z \rangle$ IS ORTHOGONAL TO $\langle 1, 2, 3 \rangle$

THEN $\langle x, y, z \rangle \cdot \langle 1, 2, 3 \rangle = 0$

$$x + 2y + 3z = 0$$

ONE SOLUTION IS $x = 1, \; y = 1, \; z = -1$: $\langle 1, 1, -1 \rangle$

(NOT THE <u>ONLY</u> ONE!)

NOW MULTIPLY BY A SCALAR TO MAKE IT A UNIT VECTOR (MAGNITUDE 1)

$$\frac{\langle 1, 1, -1 \rangle}{\sqrt{1^2 + 1^2 + (-1)^2}} = \boxed{\frac{1}{\sqrt{3}} \langle 1, 1, -1 \rangle}$$

OR $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right\rangle$

5. Consider the three points $P(1,0,1)$, $Q(-2,1,3)$, and $R(4,2,5)$.

(a) (6 points) Find a vector orthogonal to the plane containing P, Q, and R.

THE PLANE CONTAINS THE VECTORS $\overrightarrow{PQ}$ & $\overrightarrow{PR}$

SO $\overrightarrow{PQ} \times \overrightarrow{PR}$ IS ORTHOGONAL TO THE PLANE.

$\overrightarrow{PQ} = \langle -2-1, 1-0, 3-1 \rangle = \langle -3, 1, 2 \rangle$

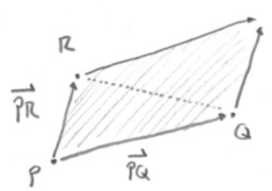
$\overrightarrow{PR} = \langle 4-1, 2-0, 5-1 \rangle = \langle 3, 2, 4 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = \langle 4-4, 6+12, -6-3 \rangle$$

$$= \boxed{\langle 0, 18, -9 \rangle}$$ OR ANY NON-ZERO SCALAR MULTIPLE.

NOTE: YOU CAN CHECK THAT THIS IS $\perp$ TO $\overrightarrow{PQ}$ & $\overrightarrow{PR}$.

(b) (4 points) Find the area of the triangle with vertices P, Q, and R.



AREA OF PARALLELOGRAM DETERMINED BY $\overrightarrow{PQ}$ & $\overrightarrow{PR}$ $= |\overrightarrow{PQ} \times \overrightarrow{PR}|$

$\therefore$ AREA OF $\Delta PQR = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2}\sqrt{0^2 + 18^2 + (-9)^2}$

$$= \frac{1}{2}\sqrt{9^2(2^2+1)} = \boxed{\frac{9\sqrt{5}}{2}}$$