

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. (8 points) Sketch the domain of the function  $f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$ .

2. (8 points) Draw a contour map of the function  $f(x, y) = xe^y$  showing at least four level curves (i.e. contour lines).

3. (8 points) Show that the following limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{5y^4 \cos^2 x}{x^4 + y^4}$$

4. (8 points) For what value of  $c$  is the following function continuous at the origin?

$$f(x, y) = \begin{cases} \frac{x^4 - y^4}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ c & \text{if } (x, y) = (0, 0) \end{cases}$$

5. (8 points) Find all second partial derivatives for the following function.

$$f(x, y) = x^3y^5 + 2x^4y$$

6. (8 points) Find an equation of the tangent plane to the surface

$$f(x, y) = \frac{2x + 3}{4y + 1}.$$

at the point  $(0, 0, 3)$ .

7. (8 points) ANSWER ONE OF THE FOLLOWING QUESTIONS.

(a) Find parametric equations for the surface obtained by rotating the curve

$$y = e^{-x}, \quad 0 \leq x \leq 3$$

about the  $x$ -axis.

(b) Use linear approximation or differentials to estimate  $\frac{10.1}{\sqrt{3.8}}$ .

*Hint: consider the function  $f(x, y) = \frac{x}{\sqrt{y}}$ .*