Name: *ANSWER KEY*

10/23/2018

Math 20300-ST Calculus III

Quiz 3

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

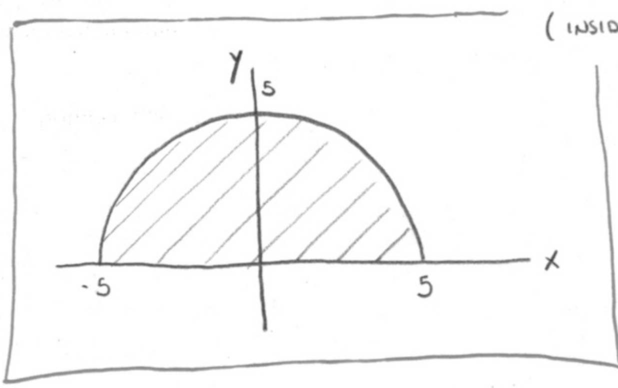
1. (8 points) Sketch the domain of the function $f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$.

EXPRESSIONS INSIDE $\sqrt{\ }$ MUST BE NON-NEGATIVE:

$y \geq 0$, (NOT BELOW X-AXIS)

$25 - x^2 - y^2 \geq 0$

$25 \geq x^2 + y^2$ (INSIDE CIRCLE OF RADIUS 5, CENTER (0,0))

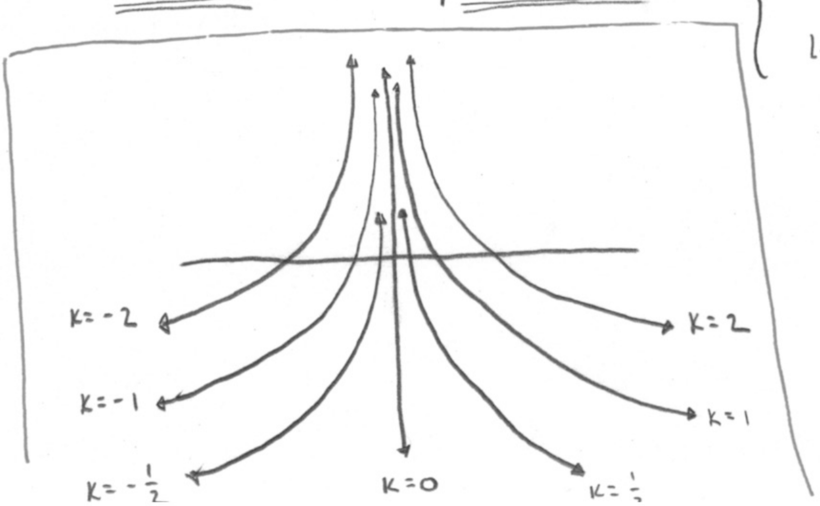


2. (8 points) Draw a contour map of the function $f(x, y) = xe^y$ showing at least four level curves (i.e. contour lines).

SETTING $f(x,y) = k$, WE GET A DIFFERENT LEVEL CURVE FOR EACH VALUE OF k IN THE RANGE OF f.

$xe^y = k$ (NOTE: IF $k > 0$ THEN $x > 0$, IF $k < 0$ THEN $x < 0$)

$\underline{\underline{x = ke^{-y}}}$ OR $y = \underline{\underline{\ln\left(\frac{k}{x}\right)}} = \begin{cases} \ln k - \ln x & \text{IF } k, x > 0 \\ \ln(-k) - \ln(-x) & \text{IF } k, x < 0 \end{cases}$



3. (8 points) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{5y^4\cos^2 x}{x^4+y^4}$$

ALONG $x=0$: $\lim\limits_{y\to 0} \dfrac{5y^4\cos^2(0)}{0^4+y^4} = \lim\limits_{y\to 0} \dfrac{5\cancel{y^4}}{\cancel{y^4}} = 5$

ALONG $y=0$: $\lim\limits_{x\to 0} \dfrac{5(0)^4\cos^2(x)}{x^4+0^4} = \lim\limits_{x\to 0} \dfrac{0}{x^4} = 0$

SINCE THE LIMIT DEPENDS ON THE PATH (x,y) TAKES ON ITS WAY TO $(0,0)$ (DIFFERENT LIMITS ALONG x & y AXES) THE LIMIT DOES NOT EXIST.

4. (8 points) For what value of c is the following function continuous at the origin?

$$f(x,y) = \begin{cases} \dfrac{x^4-y^4}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ c & \text{if } (x,y) = (0,0) \end{cases}$$

f IS CONTINUOUS AT $(0,0)$ IF $\lim\limits_{(x,y)\to(0,0)} f(x,y) = f(0,0) = c$

$\therefore$ SET $c = \lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} \underbrace{\dfrac{\cancel{(x^2+y^2)}(x^2-y^2)}{\cancel{(x^2+y^2)}}}$

NOW SET $x = r\cos\theta$
$y = r\sin\theta$

AND LET $r\to 0$

$= \lim\limits_{r\to 0} r^2 \underbrace{(\cos^2\theta - \sin^2\theta)}_{\text{ALWAYS BETWEEN } -1 \text{ \& } 1} = 0$

$\boxed{c = 0}$

5. (8 points) Find all second partial derivatives for the following function.

$$f(x, y) = x^3y^5 + 2x^4y$$

$f_x(x,y) = 3x^2y^5 + 8x^3y$

$f_y(x,y) = 5x^3y^4 + 2x^4$

$f_{xx}(x,y) = 6xy^5 + 24x^2y$

$f_{yy}(x,y) = 20x^3y^3$

$f_{xy}(x,y) = 15x^2y^4 + 8x^3$

$f_{yx}(x,y) = 15x^2y^4 + 8x^3$

SAME! ALWAYS THE SAME WHEN MIXXED PARTIALS ARE CONTINUOUS. POLYNOMIALS ARE CONTINUOUS EVERYWHERE.

6. (8 points) Find an equation of the tangent plane to the surface

$$f(x, y) = \frac{2x + 3}{4y + 1}.$$

at the point $(0, 0, 3)$.

EQ: $z = 3 + f_x(0,0)(x-0) + f_y(0,0)(y-0)$

$f_x = \dfrac{2}{4y+1} \Rightarrow f_x(0,0) = 2$

$f_y = \dfrac{\partial}{\partial y}\left[(2x+3)(4y+1)^{-1}\right] = (2x+3)(-1)(4y+1)^{-2}(4) = \dfrac{-4(2x+3)}{(4y+1)^2}$

$\Rightarrow f_y(0,0) = -12$

$\therefore \quad \boxed{z = 3 + 2x - 12y}$

7. (8 points) ANSWER ONE OF THE FOLLOWING QUESTIONS.

(a) Find parametric equations for the surface obtained by rotating the curve

$$y = e^{-x}, \qquad 0 \le x \le 3$$

about the x-axis.

IN GENERAL, $y = f(x)$ ($a \le x \le b$) ROTATED AROUND X-AXIS YIELDS A SURFACE WITH

PARAMETRIC EQUATION $\vec{r}(x, \theta) = \langle x, f(x)\cos\theta, f(x)\sin\theta \rangle$, $a \le x \le b$, $0 \le \theta \le 2\pi$

NOTE: SWITCH SIN COS IS OK TOO!

$$\therefore \boxed{\vec{r}(x,\theta) = \langle x, e^{-x}\cos\theta, e^{-x}\sin\theta \rangle, \quad 0 \le x \le 3, \quad 0 \le \theta \le 2\pi}$$

(b) Use linear approximation or differentials to estimate $\dfrac{10.1}{\sqrt{3.8}}$.

Hint: consider the function $f(x, y) = \dfrac{x}{\sqrt{y}}$.

APPROXIMATE $f(10.1, 3.8)$ WITH $L(10.1, 3.8)$ WHERE

$L(x,y)$ = LINEAR APPROXIMATION TO $f(x,y)$ AT $(10, 4)$

$$L(x,y) = f(10,4) + f_x(10,4)(x-10) + f_y(10,4)(y-4)$$

$$f_x = \frac{1}{\sqrt{y}} \rightarrow f_x(10,4) = \frac{1}{2} \quad ; \quad f_y = \frac{-x}{2y\sqrt{y}} \rightarrow f_y(10,4) = \frac{-10}{16} = -\frac{5}{8}$$

$$L(x,y) = 5 + \left(\tfrac{1}{2}\right)(x-10) - \tfrac{5}{8}(y-4)$$

$$L(10.1, 3.8) = 5 + \left(\tfrac{1}{2}\right)\left(\tfrac{1}{10}\right) - \tfrac{5}{8}\left(-\tfrac{1}{5}\right) = 5 + \tfrac{1}{20} + \tfrac{1}{8} \; \boxed{= 5\tfrac{7}{40} \text{ OR } 5.175}$$

(FYI: $\frac{10.1}{\sqrt{3.8}} = 5.181\ldots$