Name: *Answer Key* 10/12/2017

Math 205 Elements of Calculus [redacted] Exam 1

1. (8 points) Let

$$f(x) = \ln(1+\sqrt{x}), \quad g(x) = 3x^2 + \frac{2}{x+1}.$$

Find both $f(g(x))$ and $g(f(x))$. You do not need to simplify your answer.

$$f(g(x)) = \ln(1+\sqrt{g(x)}) = \boxed{\ln\left(1 + \sqrt{3x^2 + \frac{2}{x+1}}\right)}$$

$$g(f(x)) = 3f(x)^2 + \frac{2}{f(x)+1} = \boxed{3\ln(1+\sqrt{x})^2 + \frac{2}{\ln(1+\sqrt{x})+1}}$$

2. (16 points) Give an equation for the line that passes through the point $(5, -1)$ and

(a) is horizontal; $\boxed{y = -1}$

(b) is vertical; $\boxed{x = 5}$

(c) is parallel to the line $8x + 7y = 11$;

$$y = -\frac{8}{7}x + \frac{11}{7} \Rightarrow \text{slope} = -\frac{8}{7}$$

PNT-SLOPE FORMULA: $y - y_1 = m(x - x_1) \longrightarrow \boxed{y + 1 = -\frac{8}{7}(x-5) \text{ or } y = -\frac{8}{7}x + \frac{33}{7}}$

(d) also passes through the point $(-2, 1)$.

$$\text{SLOPE } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-1)}{-2-5} = -\frac{2}{7}$$

$$\therefore \boxed{y + 1 = -\frac{2}{7}(x-5) \quad \text{or} \quad y = -\frac{2}{7}x + \frac{3}{7}}$$

3. (8 points) Evaluate each of the following expressions.

(a) 3^{-4} $= \frac{1}{3^4} = \boxed{\frac{1}{81}}$

(b) $16^{3/4}$ $= \left(16^{1/4}\right)^3 = 2^3 = \boxed{8}$

(c) $\log_7(1/49)$ $= x \iff 7^x = \frac{1}{49} = \frac{1}{7^2} \Rightarrow x = \boxed{-2}$

(d) $\ln(\sqrt{e})$ $= \ln\left(e^{1/2}\right) = \boxed{\frac{1}{2}}$

4. (8 points) Suppose $\ln a = 2$, $\ln b = -3$, and $\ln c = 9$. Use log rules to evaluate

$$\ln\left(\frac{a^{\cancel{2}\,12}}{b^4c^3}\right) + \ln(e) - \ln(1).$$

$\ln(a^{12}) - \ln(b^4c^3) + 1 - 0$

$12\ln a - 4\ln b - 3\ln c + 1 = 12(2) - 4(-3) - 3(9) + 1$

$= 24 + 12 - 27 + 1 = \boxed{10}$

5. (8 points) Suppose $f(x) = Ca^x$, where C and a are both constants and $a > 0$. If $f(0) = 120$ and $f(4) = 1920$, find C and a.

$f(0) = 120$ THEN $f(4) = 1920$

$Ca^0 = 120$

$120 \cdot a^4 = 1920$

$\boxed{c = 120}$

$a^4 = \frac{1920}{120} = \frac{192}{12} = 16$

$\boxed{a = 2}$

6. (20 points) Evaluate each of the following limits.

(a) $\lim_{x\to 5^-} \frac{x-3}{5-x}$ $\begin{matrix} \to 2 \\ \to 0 \end{matrix}$ $\therefore$ LIMIT $= \pm\infty$, WHICH ONE ?

$x \to 5^- \Rightarrow x < 5 \Rightarrow 0 < 5 - x$ Pos.

$\therefore \lim_{x\to 5^-} \frac{x-3}{5-x} \begin{matrix} \text{Pos.} \\ \text{Pos.} \end{matrix} \to \boxed{\infty}$

(b) $\lim_{x\to 3^+} \frac{3-x}{x-3} = \lim_{x\to 3^+} \frac{-\cancel{(x-3)}}{\cancel{(x-3)}} = \boxed{-1}$

(c) $\lim_{t\to 4} \frac{t^2+2t-24}{t^2-5t+4} = \lim_{t\to 4} \frac{(t+6)\cancel{(t-4)}}{(t-1)\cancel{(t-4)}}$

$= \frac{4+6}{4-1} = \boxed{\frac{10}{3}}$

(d) $\lim\limits_{w\to 2} \dfrac{\frac{4}{2w+1} - \frac{4}{5}}{w-2}$ $= \lim\limits_{w\to 2} \dfrac{\dfrac{4(5) - 4(2w+1)}{5(2w+1)}}{w-2}$

$$= \lim_{w\to 2} \frac{16 - 8w}{5(2w+1)(w-2)} = -\frac{8}{5} \lim_{w\to 2} \frac{\cancel{(w-2)}}{(2w+1)\cancel{(w-2)}}$$

$$= -\frac{8}{5} \cdot \frac{1}{2(2)+1} = \boxed{-\frac{8}{25}}$$

(e) $\lim\limits_{x\to 81} \dfrac{x-81}{\sqrt{x}-9}$ $= \lim\limits_{x\to 81} \dfrac{x-81}{\sqrt{x}-9} \cdot \dfrac{\sqrt{x}+9}{\sqrt{x}+9}$

$$= \lim_{x\to 81} \frac{\cancel{(x-81)}(\sqrt{x}+9)}{\cancel{(x-81)}} = \sqrt{81} + 9 = \boxed{18}$$

7. (8 points) Let

$$f(x) = \begin{cases} \dfrac{5+\sqrt{x}}{\sqrt{5+x}} & \text{if } 0 < x < 4 \\ cx + \dfrac{1}{3} & \text{if } 4 \le x \end{cases}$$

For what value of the constant c is the function f continuous at $x = 4$?

$$\lim_{x \to 4^+} f(x) = f(4) = 4c + \frac{1}{3} \qquad (1)$$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{5+\sqrt{x}}{\sqrt{5+x}} = \frac{5+\sqrt{4}}{\sqrt{5+4}} = \frac{7}{3} \qquad (2)$$

f IS CONTINUOUS AT $x=4$ IF $(1) = (2)$

$$4c + \frac{1}{3} = \frac{7}{3}$$

$$4c = 2 \rightarrow \boxed{c = \frac{1}{2}}$$

8. (8 points) Let $f(x) = 5x^2 - 3x + 2$. Use the definition of the derivative as a limit to evaluate $f'(3)$.

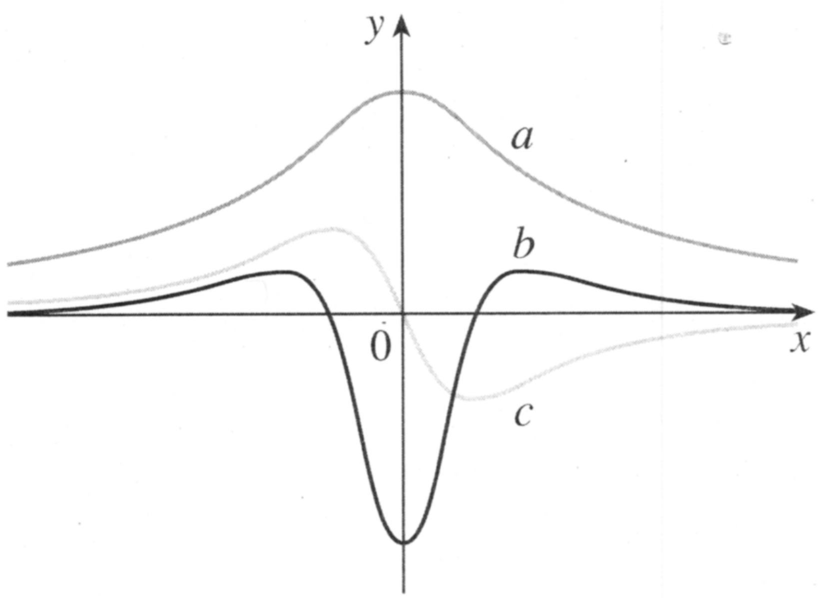
$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{5(3+h)^2 - 3(3+h) + 2 - 38}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{45} + 30h + 5h^2 - \cancel{9} - 3h + \cancel{2} - \cancel{38}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(27 + 5h)}{\cancel{h}} = \boxed{27}$$

9. (6 points) The figure below shows the graphs of f, f', and f''. The curves are labeled a, b, and c. Identify each curve.



NOTICE THAT CURVE a IS ALWAYS POSITIVE.

SINCE NEITHER OF THE OTHER CURVES ARE ALWAYS INCREASING,

CURVE a IS NOT THE DERIVATIVE OF EITHER ONE.

THEREFORE $\boxed{a = f(x)}$

a, i.e. f(x), IS INCREASING WHEN $x < 0$ & DECREASING WHEN $x > 0$

THEREFORE $f'(x)$ IS POSITIVE WHEN $x < 0$ & NEGATIVE WHEN $x > 0$.

THEREFORE $\boxed{c = f'(x)}$

SO $\boxed{b = f''(x)}$

10. (8 points) A manufacturers weekly cost, in dollars, for producing q lamps is

$$C(q) = \cancel{16000 + 500q - 1.6q^2} \quad 810 + 3q + .002q^2$$

Find the number of lamps that should be produced in order to minimize the average cost.

SOLVE $\quad C'(q) = \dfrac{C(q)}{q}$

$$3 + .004q = \frac{810 + 3q + .002q^2}{q}$$

$$\cancel{3q} + .004q^2 = 810 + \cancel{3q} + .002q^2$$

$$.002q^2 = 810$$

$$\boxed{q = \sqrt{\frac{810}{.002}} = \sqrt{810 \cdot 500} = 450\sqrt{2}}$$

11. (8 points) Find the production level that will maximize profit if the cost and demand functions are the following.

$$C(q) = 16000 + 500q - 1.6q^2. \quad + .004q^3$$

$$D(q) = 1700 - 7q$$

$R(q) = q\,D(q) = 1700q - 7q^2$

SOLVE $\quad C'(q) = R'(q) \longrightarrow 500 - 3.2q + .012q^2 = 1700 - 14q$

$.012q^2 + 10.8q - 1200 = 0$

MULTIPLY BY 1000

$12q^2 + 10800q - 1200000 = 0$

DIVIDE BY 12

$q^2 + 900q - 100000 = 0$

$(q - 100)(q + 1000) = 0$

$\boxed{q = 100}$ $\quad q = -1000$ (PRODUCTION LEVEL CANNOT BE NEGATIVE)