Name: *Answer Key* 11/16/2017

Math 205 Elements of Calculus Exam 2 Part II

1. Differentiate each of the following functions.

(a) (8 points) $F(x) = (e^{3x} + x^2)^5$

$F'(x) = 5(e^{3x} + x^2)^4 (3e^{3x} + 2x)$

(b) (8 points) $G(x) = \dfrac{(2x+1)^5}{(3x-1)^4}$

$$G'(x) = \frac{5(2x+1)^4(2)(3x-1)^4 - (2x+1)^5 4(3x-1)^3(3)}{(3x-1)^8}$$

$$= \frac{2(2x+1)^4(3x-1)^3\left[5(3x-1) - 6(2x+1)\right]}{(3x-1)^6}$$

$$= \frac{2(2x+1)^4(3x-11)}{(3x-1)^5}$$

2. (8 points) Use log rules to simplify the following expression for $f(x)$. Then find $f'(x)$.

$$f(x) = \ln\left(\frac{(x^2+1)e^x}{\sqrt{x}}\right)$$

$f(x) = \ln(x^2+1) + x - \frac{1}{2}\ln x$

$f'(x) = \frac{2x}{x^2+1} + 1 - \frac{1}{2x} = \frac{4x^2 + 2x(x^2+1) - (x^2+1)}{2x(x^2+1)}$

$= \frac{2x^3+3x^2+2x-1}{2x(x^2+1)}$

3. (12 points) Use implicit differentiation to find an equation for the tangent line to the following curve at the point $(1,2)$.

$$x^2 + 2xy - y^2 + x = 2$$

$2x + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} + 1 = 0$

$\frac{dy}{dx} = \frac{-2x-2y-1}{2x-2y} \xrightarrow[y=2]{x=1} \frac{-2(1)-2(2)-1}{2(1)-2(2)} = \frac{-7}{-2} = \frac{7}{2}$

$y - 2 = \frac{7}{2}(x-1)$ or $y = \frac{7}{2}x - \frac{3}{2}$

4. (12 points) A spherical snowball with radius r and volume $V = \frac{4}{3}\pi r^3$ begins to melt at 9am. At 10am the volume is 36π cubic inches and the volume is decreasing at a rate of 1 cubic inch per minute. What is the rate of change of the radius at that time?

FIND $\frac{dr}{dt}$ WHEN $V = 36\pi$ AND $\frac{dV}{dt} = -1$

$V = \frac{4}{3}\pi r^3$

$36\pi = \frac{4}{3}\pi r^3 \Rightarrow 27 = r^3 \Rightarrow r = 3$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

PLUG IN: $-1 = 4\pi(3)^2 \frac{dr}{dt}$

$\therefore \boxed{\frac{dr}{dt} = \frac{-1}{36\pi}}$

5. (16 points) Let $f(x) = x^4 - 4x^3$.

(a) Find the intervals on which f is increasing/decreasing.

(b) List any/all local maximums and minimums.

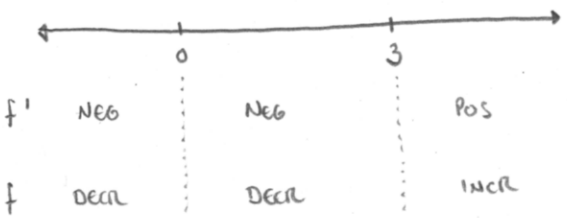
(c) Find the intervals on which f is concave up/down.

(d) List any/all inflection points for the graph $y = f(x)$.

(e) Use the information from parts (a)-(d) to sketch (roughly) the graph $y = f(x)$.

(a) $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) = 0$

CRIT. NUM.'S : $x = 0, 3$



INCREASING : $(3, \infty)$, DECREASING: $(-\infty, 3)$

(b) LOCAL MIN @ $x = 3$, $f(3) = -27$

(c) $f''(x) = 12x^2 - 24x = 12x(x-2) = 0$

$x = 0$, $x = 2$

CONCAVE UP : $(-\infty, 0) \cup (2, \infty)$

CONCAVE DOWN : $(0, 2)$

(d) P.O.I. : $(0,0)$, $(2, -16)$