Name: *ANSWER KEY* 10/5/2017

Math 205 Elements of Calculus Practice Exam 1

1. (8 points) Let

$$f(x) = \sqrt{x^2 - 1}, \quad g(x) = 2x + 5.$$

Find both $f(g(x))$ and $g(f(x))$.

$f(g(x)) = \sqrt{g(x)^2 - 1} = \boxed{\sqrt{(2x+5)^2 - 1}}$

$= \sqrt{4x^2 + 20x + 25 - 1} = \sqrt{4x^2 + 20x + 24}$

OR

$= \sqrt{4(x^2 + 5x + 6)} = \boxed{2\sqrt{x^2 + 5x + 6}}$

$g(f(x)) = 2f(x) + 5 = \boxed{2\sqrt{x^2 - 1} + 5}$

2. (16 points) Give an equation for the line that passes through the point $(-2, 3)$ and

(a) is horizontal;

$\boxed{y = 3}$

(b) is vertical;

$\boxed{x = -2}$

(c) is parallel to the line $2y - 6x = 1$; $\longrightarrow$ $y = 3x + \frac{1}{2}$ $\longrightarrow$ SLOPE 3

PNT-SLOPE FORMULA : $y - y_1 = m(x - x_1)$

$\boxed{y - 3 = 3(x + 2)}$ OR $\boxed{y = 3x + 9}$

(d) also passes through the point $(1, 1)$.

$m = \dfrac{y_2 - y_1}{x_2 - x} = \dfrac{1 - 3}{1 + 2} = \dfrac{-2}{3}$

PNT-SLOPE FORMULA : $\boxed{y - 3 = -\frac{2}{3}(x + 2)}$

OR $\boxed{y = -\frac{2}{3}x + \frac{5}{3}}$

3. (8 points) Evaluate each of the following expressions.

(a) $36^{-1/2}$ $= \frac{1}{36^{1/2}} = \frac{1}{\sqrt{36}} = \boxed{\frac{1}{6}}$

(b) $64^{2/3}$ $= (64^{1/3})^2 = (\sqrt[3]{64})^2 = 4^2 = \boxed{16}$

(c) $\log_2(1/16)$ $= x$ MEANS $2^x = \frac{1}{16} = \frac{1}{2^4} = 2^{-4}$ $\Rightarrow x = \boxed{-4}$

(d) $\ln(e^7)$ $= \boxed{7}$

4. (8 points) Suppose $\ln a = 4$, $\ln b = -5$, and $\ln c = 6$. Use log rules to evaluate

$$\ln\left(\frac{a^8}{b^3c^5}\right).$$

$= \ln(a^8) - \ln(b^3c^5) = \ln(a^8) - [\ln(b^3) + \ln(c^5)]$

$= 8\ln a - 3\ln b - 5\ln c = 8(4) - 3(-5) - 5(6)$

$= 32 + 15 - 30 = \boxed{17}$

5. (8 points) Suppose $f(x) = Ca^x$, where C and a are both constants and $a > 0$. If $f(0) = 500$ and $f(3) = 8000$, find C and a.

$f(0) = 500 \Rightarrow Ca^0 = 500 \Rightarrow \boxed{C = 500}$

$f(3) = 8000 \Rightarrow 500a^3 = 8000$

$a^3 = 16 \Rightarrow \boxed{a = 16^{1/3}}$

$\therefore f(x) = 500(16^{1/3})^x$ i.e. $f(x) = 500 \cdot 16^{x/3}$

6. (20 points) Evaluate each of the following limits.

(a) $\lim_{x\to 2^-} \frac{x-3}{x-2}$ $\Rightarrow$ $\frac{\text{NEG} \to -1}{\text{NEG} \to 0} = \boxed{\infty}$

$\hookrightarrow$ $x < 2 \Rightarrow x-2 < 0$

$x - 3 < 0$

(b) $\lim_{x\to 2^+} \frac{x-3}{x-2}$ $\Rightarrow$ $\frac{\text{NEG} \to -1}{\text{POS} \to 0} = \boxed{-\infty}$

$\hookrightarrow$ $x > 2 \Rightarrow x-2 > 0$

$x - 3 < 0$

(c) $\lim_{t\to -1} \frac{t^2-3t-4}{t+1} = \lim_{t\to -1} \frac{(t-4)(t+1)}{t+1} = \lim_{t\to -1} t-4 = \boxed{-5}$

(d) $\lim_{w\to 3} \frac{\frac{1}{w}-\frac{1}{3}}{w-3} = \lim_{w\to 3} \frac{\frac{3}{3w} - \frac{w}{3w}}{w-3} = \lim_{w\to 3} \frac{3-w}{3w(w-3)}$

$= \lim_{w\to 3} \frac{1}{3w} \cdot \underbrace{\frac{3-w}{w-3}}_{-1} = \lim_{w\to 3} -\frac{1}{3w} = \boxed{-\frac{1}{9}}$

(e) $\lim_{x\to 0} \frac{\sqrt{x^2+c}-\sqrt{c}}{x^2} \cdot \frac{\sqrt{x^2+c}+\sqrt{c}}{\sqrt{x^2+c}+\sqrt{c}} = \lim_{x\to 0} \frac{x^2 + \cancel{c} - \cancel{c}}{x^2(\sqrt{x^2+c}+\sqrt{c})}$ 1

$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+c}+\sqrt{c}} = \boxed{\frac{1}{2\sqrt{c}}}$

7. (8 points) Let

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } 2 \le x \end{cases}$$

For what value of the constant c is the function f continuous everywhere?

BOTH $cx^2 + 2x$ AND $x^3 - cx$ ARE POLYNOMIALS, AND SO f IS CONTINUOUS EVERYWHERE EXCEPT POSSIBLE AT $x = 2$, WHERE WE NEED $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$$\lim_{x \to 2^-} cx^2 + 2x = \lim_{x \to 2^+} x^3 - cx$$

$$4c + 4 = 8 - 2c$$

$$6c = 4$$

$$\boxed{c = \frac{2}{3}}$$

$f(2) = 2$

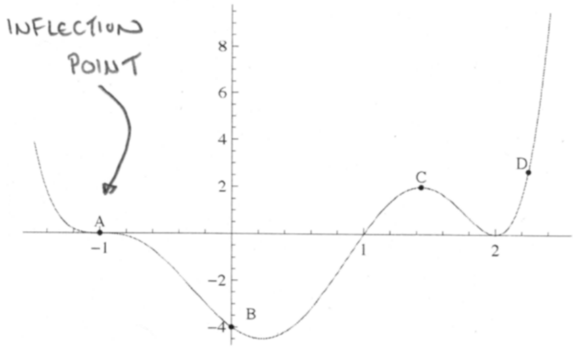
8. (8 points) Let $f(x) = 2x^2 - 3x$. Use the definition of the derivative as a limit to evaluate $f'(2)$.

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{2(2+h)^2 - 3(2+h) - 2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8} + 8h + 2h^2 \cancel{-6} - 3h \cancel{-2}}{h}$$

$$= \lim_{h \to 0} \frac{5h + 2h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(5 + 2h)}{\cancel{h}} = \boxed{5}$$

9. (12 points) Below is the graph of a function $y = f(x)$. Fill in the chart with $+$, $-$ or 0 to indicate whether f, f' and f'' are positive, negative or zero at each of the indicated points A,B,C and D.



Point	f	f'	f''
A	0	0	0
B	−	−	+
C	+	0	−
D	+	+	+

QUESTIONS TO ASK :

f : IS f ABOVE THE X-AXIS (+) OR BELOW THE X-AXIS (−)

f' : IS f INCREASING (+) OR DECREASING (−)

f'' : IS f CONCAVE UP (+) OR CONCAVE DOWN (−)

10. (8 points) A manufacturers weekly cost, in dollars, for producing q lamps is

$$C(q) = 810 + 3q + 0.002q^2.$$

Find the number of lamps that should be produced in order to minimize the average cost.

COST IS MINIMIZED WHEN MARGINAL COST = AVERAGE COST

$$\Rightarrow C'(q) = \frac{C(q)}{q} \Rightarrow 3 + .004q = \frac{810 + 3q + .002q^2}{q}$$

$$\Rightarrow \cancel{3q} + .004q^2 = 810 + \cancel{3q} + .002q^2$$

$$.002q^2 = 810$$

$$q^2 = 810 \cdot 500 = 81 \cdot 100 \cdot 25 \cdot 2$$

$$q = \pm\sqrt{(81)(100)(25)(2)} = (9)(10)(5)\sqrt{2} = \boxed{450\sqrt{2}}$$

BECAUSE $q \geq 0$

11. (8 points) Find the production level that will maximize profit if the cost and demand functions are the following.

$$C(q) = 680 + 4q + 0.01q^2$$
$$D(q) = 12 - 0.002q$$

PROFIT IS MAXIMIZED WHEN MARGINAL COST = MARGINAL REVENUE

(NOTE: REVENUE $R(q) = D(q)q = (12 - .002q)q = 12q - .002q^2$)

$$\Rightarrow C'(q) = R'(q) \Rightarrow 4 + .02q = 12 - .004q$$

$$.016q = 8$$

$$\boxed{q = 500}$$