Name:____ **11/16/2017**

Math 205 Elements of Calculus **Practice Exam 2**

1. Find the derivative of the following functions.

   (a) $F(x) = (4x^5 + \sqrt{x})(5x^2 + 2x + 4 + \frac{1}{x})$

   (b) $G(x) = \frac{2\sqrt[3]{x} + 3x^4}{4x + 3}$

   (c) $H(x) = e^x - 2^x$

   (d) $I(x) = \ln x$

2. Find the derivative of the following functions.

(a) $P(x) = (2x^4 - 8x^2 + 1)^8$

(b) $Q(x) = \sqrt{3x^2 + x}$

(c) $R(x) = \ln(4x^2 + 5x^{-2})$

(d) $S(x) = e^{2x^2+6x-1} + 5^{4x^3}$

3. For each of the following equations, find $\dfrac{dy}{dx}$.

   (a) $x^2 - y^6 = 2$

   (b) $x^4 + x^2y^3 - y = 7$

4. Give an equation for the tangent line to the curve

$$e^{xy} = x^2 + y^2$$

   at the point $(0, 1)$.

5. Suppose you deposit \$1400 into a savings account with an annual interest rate of 4.5% that is compounded quarterly.
   (a) How much will your savings be worth after 3 years?

   (b) How long will it take you savings to double?

6. Suppose a population of bacteria doubles every 2 hours. If the population at 5pm is 2400, what was the population at noon (5 hours earlier)?

7. Suppose a sample of radioactive material is observed to decay to 72% of its original mass after 18 years. Find the half-life of this material.

8. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

9. A water tank has the shape of an inverted circular cone (the "tip" of the cone is at the bottom) with a base radius of 5 ft and a height of 14 ft. Water is being pumped into the tank at a rate of 25 ft^3/min. At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft? *Hint: the volume of a cone is given by* $V = \frac{1}{3}\pi r^2 h$.

10. Find the critical numbers for the following functions.

(a) $f(x) = x \ln x$

(b) $g(x) = \sqrt{1 - x^2}$

11. Find the absolute maximum and minimum values of

$$f(x) = \frac{x}{x^2 - x + 1}$$

over the closed interval $0 \leq x \leq 3$.

12. Let $f(x) = \ln(x^4 + 27)$.

(a) Find the intervals on which f is increasing/decreasing.

(b) List any/all local maximums and minimums.

(c) Find the intervals on which f is concave up/down.

(d) List any/all inflection points for the graph $y = f(x)$.

(e) Use the information from parts (a)-(d) to sketch (roughly) the graph $y = f(x)$.