MATH 212 - DRAFT FINAL EXAM B

SOLUTIONS BY JOHN ADAMSKI

1. (a) $\displaystyle\int_0^{\pi/2} t^2 \cos t \, dt$ $\qquad u = t^2 \qquad v = \sin t$

$du = 2t\,dt \qquad dv = \cos t\,dt.$

$$= uv\Big|_0^{\pi/2} - \int_0^{\pi/2} v\,du = t^2 \sin t\Big|_0^{\pi/2} - \int_0^{\pi/2} 2t \sin t\,dt$$

$u = 2t \qquad v = -\cos t$

$du = 2dt \qquad dv = \sin t\,dt$

$$= t^2 \sin t\Big|_0^{\pi/2} - \left(\underbrace{-2t\cos t\Big|_0^{\pi/2}}_{0} + \int_0^{\pi/2} 2\cos t\,dt \right)$$

$$= \left(\frac{\pi}{2}\right)^2 - 2\sin t\Big|_0^{\pi/2} = \left(\frac{\pi}{2}\right)^2 - 2 = \boxed{\frac{\pi^2 - 8}{4}}$$

(b) $\displaystyle\int \sec^3 t \tan^3 t\,dt = \int \underbrace{\sec^2 t\,(\sec^2 t - 1)}_{u^2(u^2-1)} \underbrace{\sec t \tan t\,dt}_{du}$

$$\rightsquigarrow \int (u^4 - u^2)\,du = \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

$$\rightsquigarrow \boxed{\frac{1}{5}\sec^5 t - \frac{1}{3}\sec^3 t + C}$$