MARKED INCORRECT JUST BECAUSE WRONG VARIABLE - EMAIL ME. ~~DIFFERENT FORM~~

QUESTION 2:

$$\left[\cosh(ax) + \sinh(ax) \right]^n$$

$$\left(\frac{e^{ax} + \cancel{e^{-ax}}}{2} + \frac{e^{ax} - \cancel{e^{-ax}}}{2} \right)^n$$

$$(e^{ax})^n = \boxed{e^{nax}}$$

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§8.4 Trig Substitution

SUMMARY:

EXPRESSION	SUBSTITUTION
$\sqrt{a^2+x^2}$	$x = a\tan\theta$
$\sqrt{a^2-x^2}$	$x = a\sin\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$

THESE TRIG SUBSTITUTIONS TURN THE RADICAND ($\sqrt{}$) INTO A PERFECT SQUARE!

"REVERSE u - SUBSTITUTE"

ex. $$\int \frac{\sqrt{1-(\ln x)^2}}{x\ln x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

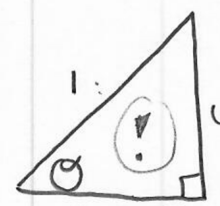
$$\rightsquigarrow \int \frac{\sqrt{1-u^2}}{u}\,du$$

u-SUB   TRIG. SUB.

LET $u = \sin\theta \xrightarrow{②}$

$\downarrow ①$

$\underline{du = \cos\theta\, d\theta}$



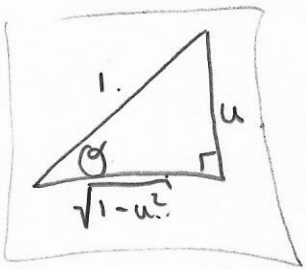
$\underline{\underline{\sqrt{1-u^2} = \cos\theta}}$

$$\rightsquigarrow \int \frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\cos\theta\, d\theta = \int \frac{\cos\theta}{\sin\theta}\cos\theta\, d\theta$$

$$= \int \frac{\cos^2}{\sin\theta}\,d\theta = \int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta = \int \csc\theta - \sin\theta\, d\theta$$

$$\left(\text{NOTE: } \begin{aligned} \int \sec\theta \, d\theta &= \ln\left|\sec\theta + \tan\theta\right| + C \\ \int \csc\theta \, d\theta &= -\ln\left|\csc\theta + \cot\theta\right| + C \end{aligned} \right)$$

$$= -\ln\left|\csc\theta + \cot\theta\right| + \cos\theta + C \qquad (\text{BACK SUBSTITUTE})$$



$u = \sin\theta$

$$\rightsquigarrow = -\ln\left|\frac{1}{u} + \frac{\sqrt{1-u^2}}{u}\right| + \sqrt{1-u^2} + C$$

$$\rightsquigarrow = \boxed{-\ln\left|\frac{1}{\ln x} + \frac{\sqrt{1-(\ln x)^2}}{\ln x}\right| + \sqrt{1-(\ln x)^2} + C}$$

ex. $\displaystyle\int \frac{5}{\sqrt{25x^2 - 9}} \, dx$ $\qquad u = 5x$

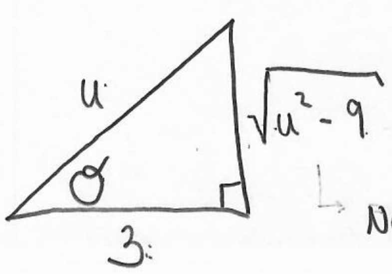
$\qquad du = 5\,dx$

$$\rightsquigarrow \int \frac{1}{\sqrt{u^2-9}} \, du$$

LET $u = 3\sec\theta$

$\downarrow$

$du = 3\sec\theta\tan\theta \, d\theta$



$\sec\theta = \dfrac{u}{3}$

NOTE: $\tan\theta = \dfrac{\sqrt{u^2-9}}{3} \Rightarrow \sqrt{u^2-9} = 3\tan\theta$

$$\rightsquigarrow \int \frac{1}{3\cancel{\tan\theta}} \cdot 3\sec\theta\, \cancel{\tan\theta}\, d\theta$$

$$= \int \sec\theta\, d\theta = \ln\left| \sec\theta + \tan\theta \right| + C$$

$$\rightsquigarrow \ln\left| \frac{u}{3} + \frac{\sqrt{u^2-9}}{3} \right| + C$$

$$\rightsquigarrow \boxed{\ln\left| \frac{5x}{3} + \frac{\sqrt{25x^2-9}}{3} \right| + C}$$

TRY ASSIGNMENT §8.4

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§8.5 INTEGRATION OF RATIONAL FUNCTIONS BY PARTIAL FRACTIONS

WE WILL LEARN HOW TO INTEGRATE **ANY** RATIONAL FUNCTION.

- RATIONAL NUMBERS $\mathbb{Q}$ (QUOTIENT) "FRACTIONS"

  $\mathbb{Q} = \left\{ \frac{a}{b} \;\middle|\; a, b \text{ INTEGERS} \right\}$

- RATIONAL FUNCTIONS $= \left\{ \frac{p(x)}{q(x)} \;\middle|\; p, q \text{ ARE POLYNOMIALS} \right\}$

e.g. $\frac{x^4 - 2x + 3}{7x^2 + 4}$ ✓ $\frac{x^2+1}{\sqrt{x+2}}$ ✗ POLY DON'T HAVE ROOTS!

THE IDEA:

$$\int \frac{1}{x^2+3x+2}\,dx$$

$\cancel{u = x + \frac{3}{2}}$

$\cancel{du = dx}$

$\cancel{x^2+3x+2}$

$\cancel{\left(x+\frac{3}{2}\right)^2 - \frac{1}{4}}$

$\cancel{\int \frac{1}{u^2-\frac{1}{4}}}$

NOTE: $$\frac{1}{x+1} - \frac{1}{x+2} = \frac{(\cancel{x}+2)-(\cancel{x}+1)}{(x+1)(x+2)}$$

$$= \frac{1}{(x+1)(x+2)} = \frac{1}{x^2+3x+2}$$

PARTIAL FRACTION DECOMPOSITION

- HOW?

$$= \int \frac{1}{x+1} - \frac{1}{x+2}\,dx = \boxed{\ln|x+1| - \ln|x+2| + C}$$

PARTIAL FRACTION DECOMPOSITION

$$\int \frac{P(x)}{Q(x)}\,dx$$, P & Q ARE POLYNOMIALS.

P.F.D. ONLY WORKS ON PROPER RATIONAL FUNCTIONS

POLY $\to$ $\frac{P(x)}{Q(x)}$ DEG (P) < DEG (Q)

(HIGHEST EXP)

IF NOT: POLYNOMIAL LONG DIVISION FIRST.

ex.

$$\int \frac{x^4+1}{x^2+2x-3}\,dx$$

$\text{DEG}(x^4+1) = 4$

$\text{DEG}(x^2+2x-3) = 2$

NOT PROPER = IMPROPER

$$\begin{array}{r}
x^2 - 2x + 7 \\
x^2+2x-3 \,\big)\, \overline{x^4 + 0x^3 + 0x^2 + 0x + 1} \\
-\left(x^4 + 2x^3 - 3x^2\right) \\
\hline
-2x^3 + 3x^2 + 0x + 1 \\
-\left(-2x^3 - 4x^2 + 6x\right) \\
\hline
7x^2 - 6x + 1 \\
-\left(7x^2 + 14x - 21\right) \\
\hline
-20x + 22
\end{array}$$

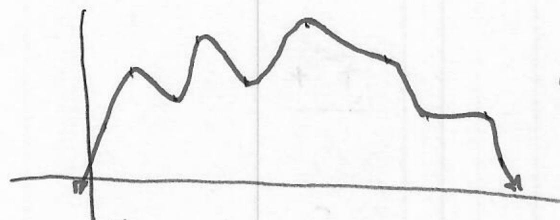
REMINDER

$$\int \frac{x^4+1}{x^2+2x-3}\,dx = \int x^2 - 2x + 7 + \frac{-20x+22}{x^2+2x-3}\,dx$$

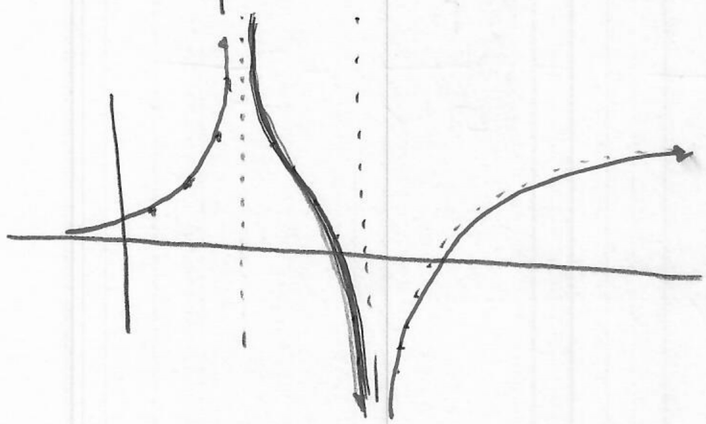
$$= \frac{1}{3}x^3 - x^2 + 7x + \int \frac{-20x+22}{x^2+2x-3}\,dx$$

PROPER

ANY CONTINUOUS FUNCTION CAN BE APPROXIMATED BY POLYNOMIALS



POLYNOMIAL



RATIONAL FUNC.

AND FUNCTIONS WITH ASYMPTOTES CAN BE APPROXIMATED

BY RATIONAL FUNCTIONS.