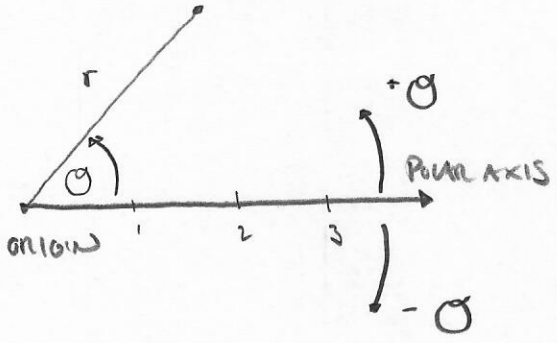
§11.3 Polar Coordinates



(r, θ)

$\left(3, \frac{7\pi}{6}\right)$

$\theta = \frac{\pi}{6}$

$\left(-3, \frac{\pi}{6} + n2\pi\right)$

$\left(-3, \frac{\pi}{6}\right)$

$\left(3, \frac{7\pi}{6}\right)$

$\left(3, \frac{7\pi}{6} + n2\pi\right)$

FULL ROTATION

$\theta = \frac{7\pi}{6}$

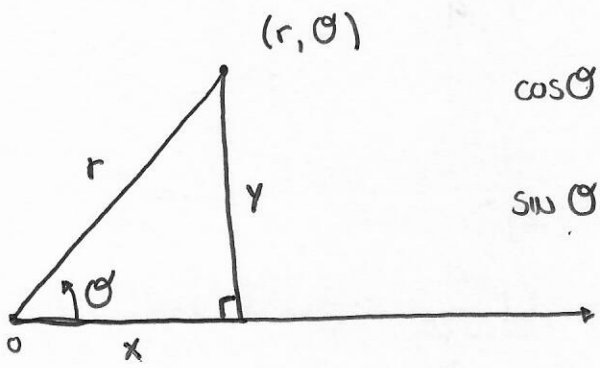
$r = 3$

$|r|$ = DISTANCE TO ORIGIN

$r = \sin\theta$ POLAR GRAPH OF EQ

UNIQUE

CONVERTING FROM POLAR TO CARTESIAN



$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$

NOT UNIQUE

CONVERTING FROM CARTESIAN TO POLAR COORD.

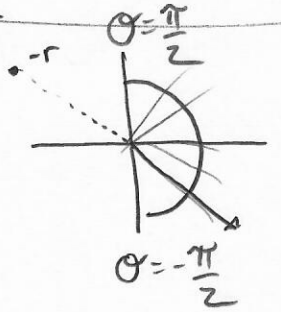
PYTH. THM. $x^2 + y^2 = r^2 \rightarrow r = \pm\sqrt{x^2 + y^2}$

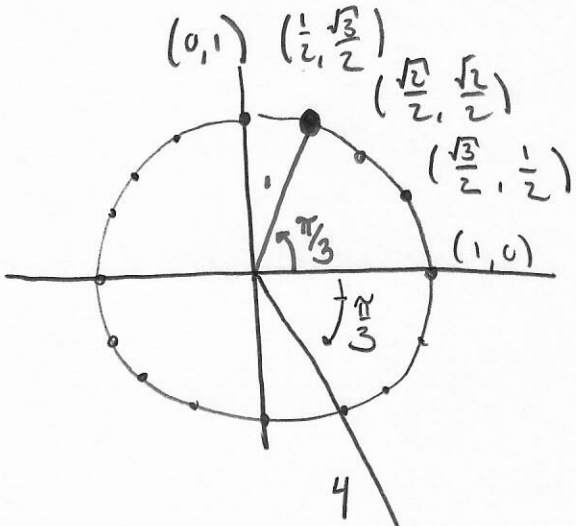
$x = r\cos\theta$

$y = r\sin\theta$

$\tan\theta = \frac{y}{x} \rightarrow \theta = \tan^{-1}\frac{y}{x}$

$-\frac{\pi}{2} < \tan(\tan^{-1}) < \frac{\pi}{2}$





$x = r\cos\theta$

$2 = 4\cos\theta$

$\frac{1}{2} = \cos\theta$

$\frac{\pi}{3}, -\frac{\pi}{3}, \frac{5\pi}{3}, -\frac{5\pi}{3}$

$x^2 + y^2 = r^2$

$(2)^2 + (-2\sqrt{3})^2 = r^2$

$4 + 12 = r^2$

$16 = r^2$

$r = 4$

CARTESIAN → POLAR

$(2, -2\sqrt{3}) \rightarrow (4, -\frac{\pi}{3}) + \pi$

$(4, -\frac{\pi}{3} + n2\pi) \rightarrow (4, -\frac{\pi}{3})$

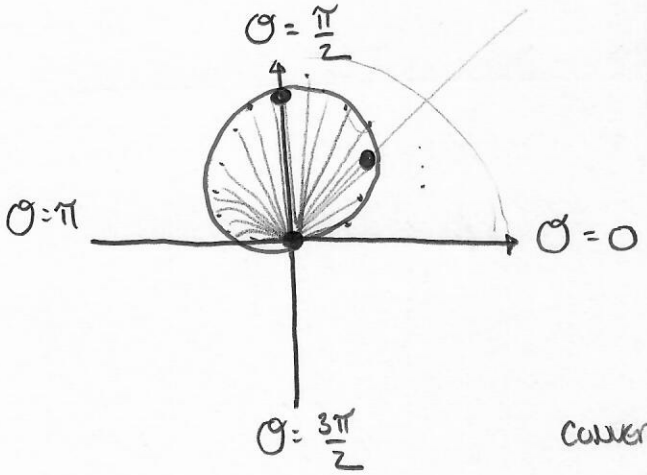
$(-4, \frac{2\pi}{3} + n2\pi) \rightarrow (-4, \frac{2\pi}{3})$

$n = 0, \pm1, \pm2, \ldots$

A **POLAR GRAPH** OF AN EQUATION $\left(r = f(\theta)\right)$

IS THE SET OF ALL POINTS IN THE PLANE WITH POLAR COORD (r, θ) THAT SATISFY THE GIVEN EQUATION.

ex. $r = \sin\theta$



PLOT:

r	θ	
$(0,$	$0)$	$0 = \sin 0$ ✓
$(1,$	$\frac{\pi}{2})$	$1 = \sin\frac{\pi}{2}$ ✓
$(\frac{\sqrt{2}}{2},$	$\frac{\pi}{4})$	$\frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}$

CONVERT THE POLAR EQ TO CARTESIAN EQ.

$$r = \sin\theta$$

ASUME $r \neq 0$: $\quad r^2 = r\sin\theta$

TOOLS

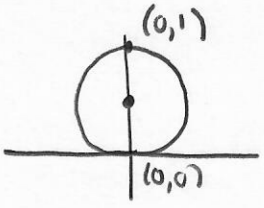
$$r^2 = x^2 + y^2$$
$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$x^2 + y^2 = y$$

$$x^2 + (y^2 - y) = 0$$

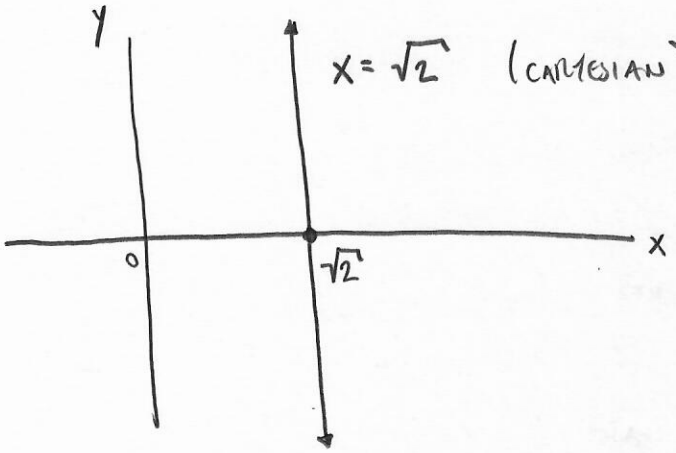
$$x^2 + \left(y - \tfrac{1}{2}\right)^2 - \tfrac{1}{4} = 0$$

$$x^2 + \left(y - \tfrac{1}{2}\right)^2 = \left(\tfrac{1}{2}\right)^2$$



CIRCLE: CENTER $(0, \frac{1}{2})$, RADIUS $\frac{1}{2}$

CONVERT CARTESIAN EQ TO POLAR EQ

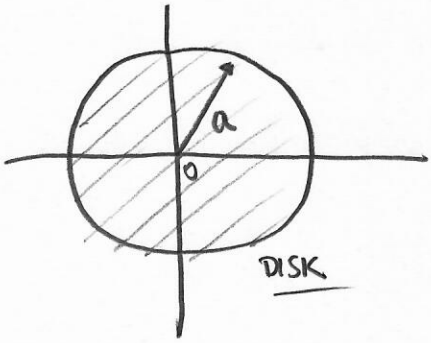


$x = \sqrt{2}$ (CARTESIAN) $\longrightarrow$ $r\cos\theta = \sqrt{2}$ (POLAR)

$r = \sqrt{2}\,\frac{1}{\cos\theta}$

$r = \sqrt{2}\sec\theta$.

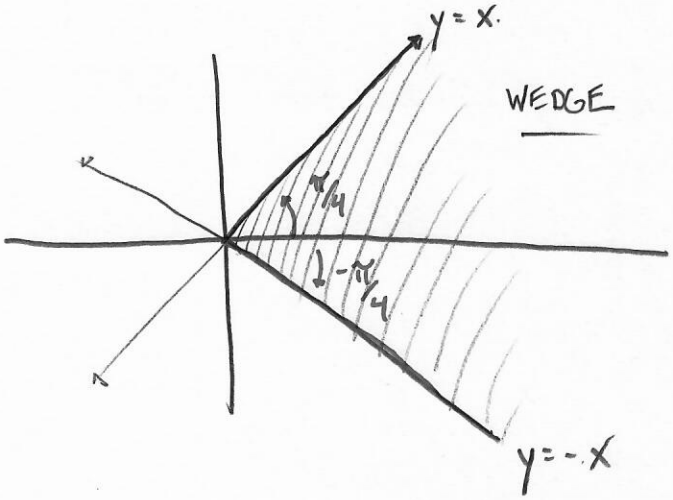
DESCRIBE REGIONS OF THE PLANE USING POLAR COORD.



CARTESIAN: $x^2 + y^2 \leq a^2$ DISK

POLAR: $0 \leq r \leq a$

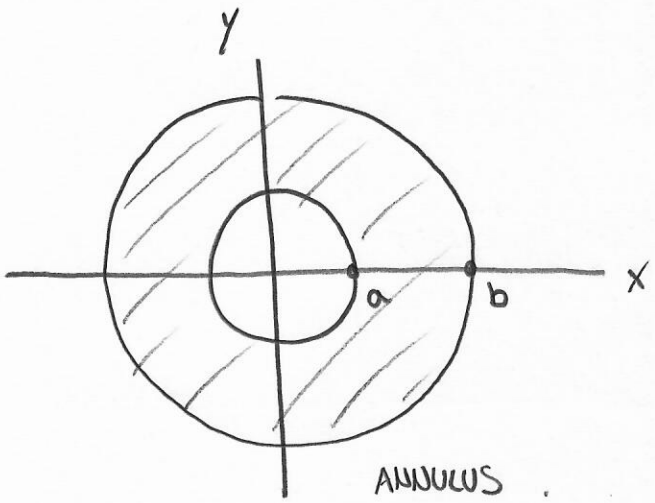
INTERVAL!



CARTESIAN: $-x \leq y \leq x$, $x \geq 0$

POLAR: $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

INTERVAL!

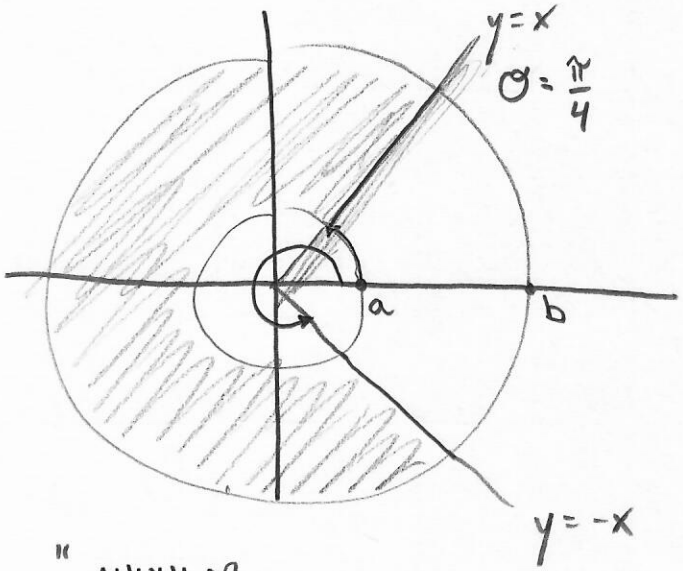


$$a^2 \le x^2 + y^2 \le b^2$$

$$a \le r \le b$$

ANNULUS.

"WASHER".



$$a \le r \le b$$

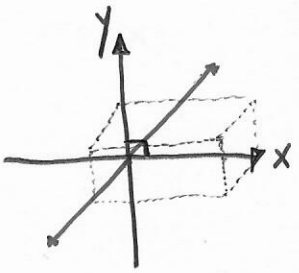
$$\frac{\pi}{4} \le \theta \le \frac{7\pi}{4}$$

"ANNULAR SECTOR"?

$$\theta = -\frac{\pi}{4}, \theta = \frac{7\pi}{4}, \ldots$$

3 DIMENSIONAL ANALYTIC GEOMETRY

§12.1 3 DIMENSIONAL COORD. SYSTEM



ADD DIMENSION $\perp$ TO BOTH x & y DIRECTIONS

i.e. $\perp$ TO THE PAGE.

WHICH WAY SHOULD WE ORIENT THIS NEW COORD AXIS (z-AXIS)

RIGHT-HAND RULE (CONVENTION)

1. GRIP THE z-AXIS WITH YOUR RIGHT HAND SO THAT YOUR FINGER CURL FROM THE (+) x-AXIS TO THE (+) y-AXIS. THEN YOUR THUMB POINTS IN THE DIRECTION OF THE POSITIVE z-AXIS.

2. POINT YOUR RIGHT INDEX FINGER IN DIRECTION OF (+) x-AXIS.
   .. .. -. POINTER .. - . . (+) y-AXIS.
   YOUR RIGHT THUMB POINTS IN THE DIRECTION OF THE POSITIVE z-AXIS.

("CHIRALITY" DIFFERENCE BETWEEN R/L HANDS)