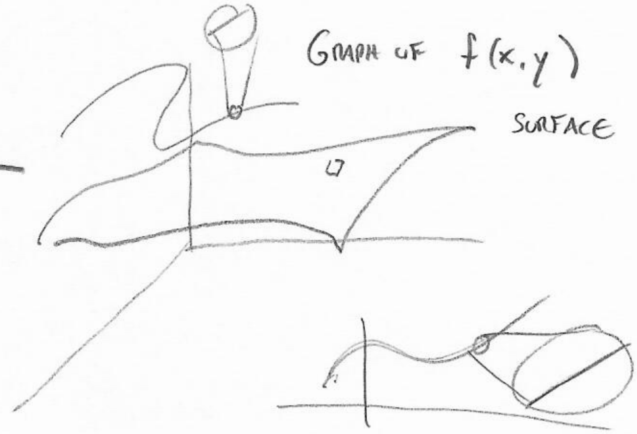
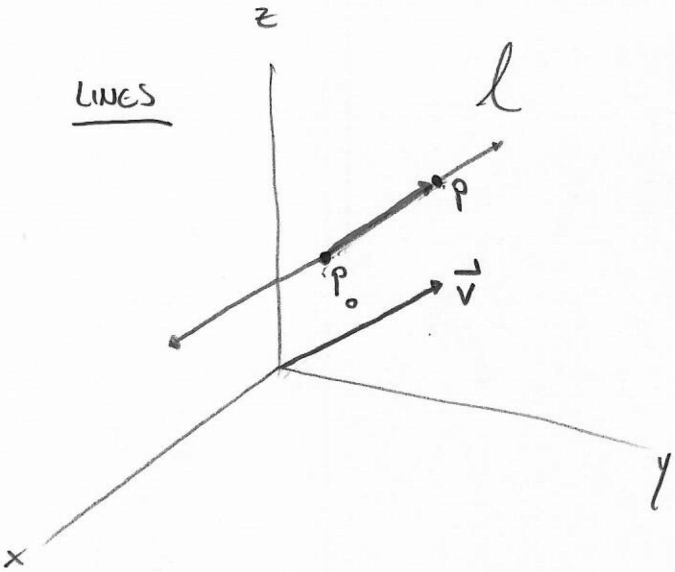
§ 12.5 EQUATIONS OF LINES & PLANES



LINES



2D: POINT P_0, SLOPE

3D: POINT P_0, DIRECTION

↑

IN 3D, DIRECTION DETERMINED BY VECTOR.

SAY WE HAVE A LINE THROUGH $P_0(x_0, y_0, z_0)$,

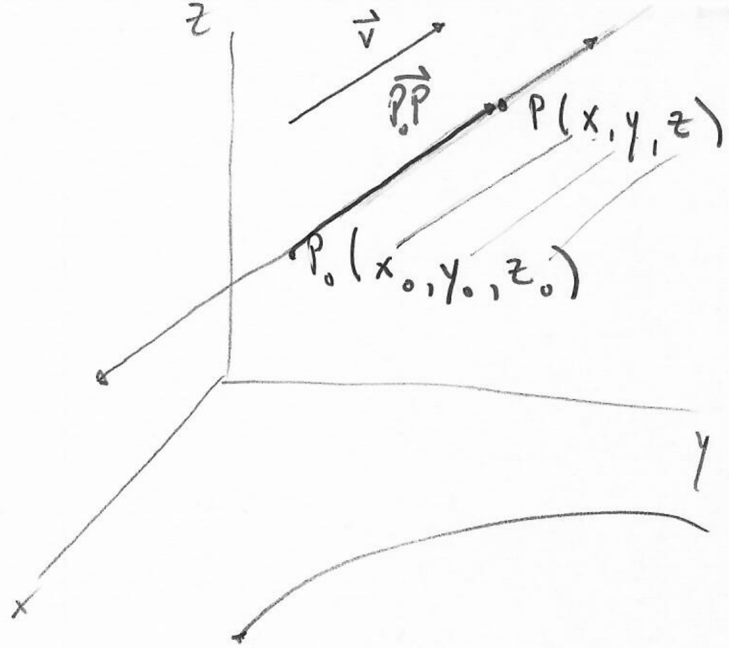
PARALLEL TO (IN THE DIRECTION OF) $\vec{v} = \langle a, b, c \rangle$

IF $P(x,y,z)$ IS ANY POINT $(P \neq P_0)$ ON THE LINE

THEN $\overrightarrow{P_0P}$ IS $\parallel$ TO $\vec{v}$

$$\Rightarrow \underbrace{\overrightarrow{P_0P} = t\vec{v}}_{\text{VECTOR EQUATION}} \quad \text{FOR SOME } t \text{ (SCALAR)}$$

FACT: 2 VECTORS ARE EQUAL $\Leftrightarrow$ THEIR VECTOR COMPONENTS ARE EQUAL.



$$\vec{P_0P} = t\vec{v}$$

$$\langle x - x_0, y - y_0, z - z_0 \rangle = t\langle a, b, c \rangle$$

$$\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle = t\langle a, b, c \rangle$$

$$\boxed{\begin{array}{ll} \langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle & \\ \vec{r}(t) = \vec{P_0} + t\vec{v} & \text{VECTOR EQ. OF A LINE} \end{array}}$$

STANDARD POSITION VECTOR FOR THE POINT P_0

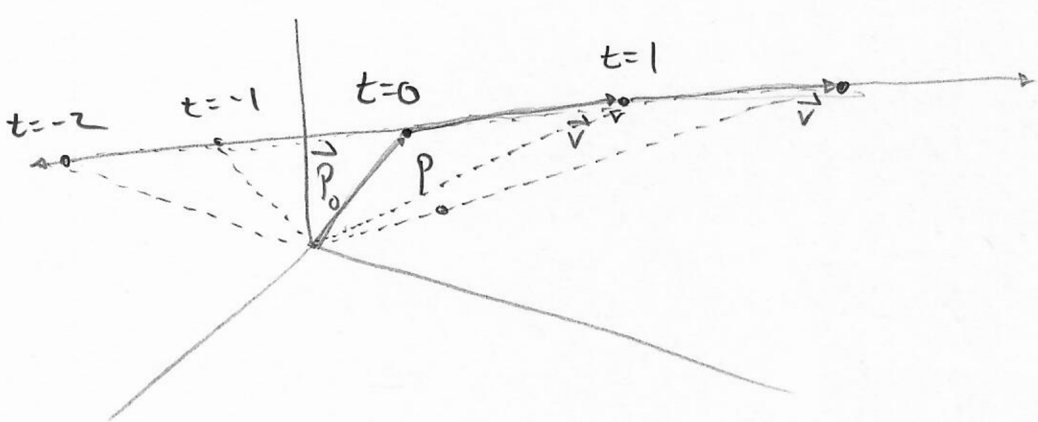
$t=0$: $\vec{r}(0) = \langle x_0, y_0, z_0 \rangle$

$t=1$: $\vec{r}(1) = \langle x_0, y_0, z_0 \rangle + \overbrace{\langle a, b, c \rangle}^{\vec{v}}$

$$(x_0, y_0, z_0) \mapsto \langle x_0, y_0, z_0 \rangle$$

STANDARD POSITION

IF INITIAL POINT IS $(0,0,0)$ THEN TERMINAL POINT IS (x_0, y_0, z_0)



2 VECTORS ARE EQUAL $\Longleftrightarrow$ COMPONENTS ARE EQUAL.

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$

3 EQ'S:

$$\boxed{\left.\begin{array}{l} x = x_0 + ta \\ y = y_0 + tb \\ z = z_0 + tc \end{array}\right\} \text{PARAMETRIC EQ OF A LINE}}$$

REVISIT PARAM. EQ'S IN 2 VAR.

ex. $x = a\cos t$

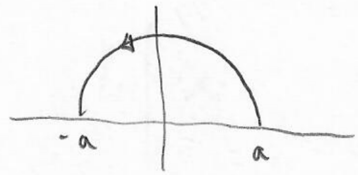
$y = a\sin t$

$0 \le t \le \pi$

} PARAMETRIC EQ.

$\rightsquigarrow \langle x, y \rangle = \langle a\cos t, a\sin t \rangle$

$\vec{r}(t) = \langle a\cos t, a\sin t \rangle$



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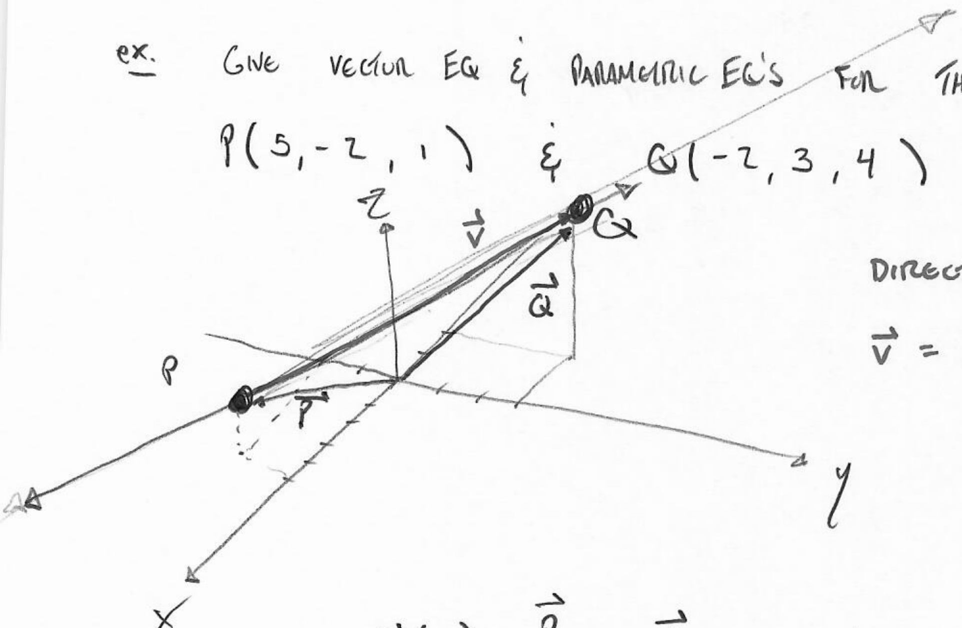
ex. GIVE VECTOR EQ & PARAMETRIC EQ'S FOR THE LINE THROUGH

$P(5,-2,1)$ & $Q(-2,3,4)$



DIRECTION OF LINE

$\vec{v} = \overrightarrow{PQ} = \vec{Q} - \vec{P}$

$= \langle -2, 3, 4 \rangle - \langle 5, -2, 1 \rangle$

$= \langle -7, 5, 3 \rangle$

$\vec{r}(t) = \vec{P}_0 + t\vec{v} = \langle 5, -2, 1 \rangle + t\langle -7, 5, 3 \rangle$

$\uparrow$ EITHER $\underline{\underline{\vec{P}}}$ OR $\vec{Q}$

$$\vec{r}(t) = \langle 5, -2, 1 \rangle + t\langle -7, 5, 3 \rangle$$

$x = 5 - 7t$

$y = -2 + 5t$

$z = 1 + 3t$

NOTE: IF WE WANTED THE LINE SEGMENT FROM P TO Q:

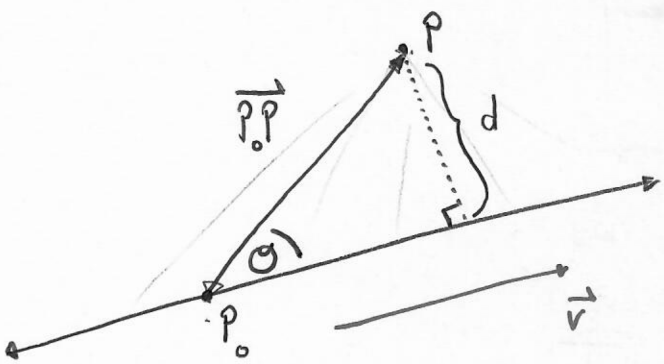
$\vec{r}(0) = \langle 5, -2, 1 \rangle = \vec{P}$

$\vec{r}(1) = \langle -2, 3, 4 \rangle = \vec{Q}$

$\Rightarrow$ RESTRICT PARAM

$0 \le t \le 1$

DISTANCE FROM A POINT TO A LINE. (d)



$$d = |\overrightarrow{P_0P}| \sin\theta$$

$$d = \frac{|\overrightarrow{P_0P}||\vec{v}| \sin\theta}{|\vec{v}|}$$

$$\therefore \boxed{d = \frac{|\overrightarrow{P_0P} \times \vec{v}|}{|\vec{v}|}}$$

P_0 = POINT ON LINE
P = POINT IN SPACE
$\vec{v}$ = DIRECTION OF LINE

ex. FIND THE DISTANCE FROM THE ORIGIN $P(0,0,0)$ TO THE LINE

$x = 2 + 3t \quad , \quad y = 4t \quad , \quad z = -2 - t$

i.e. $\langle x, y, z\rangle = \langle 2 + 3t, 4t, -2 - t\rangle$

$$\langle x, y, z\rangle = \underbrace{\langle 2, 0, -2\rangle}_{P_0} + t\underbrace{\langle 3, 4, -1\rangle}_{\vec{v}}$$

$$\overrightarrow{P_0P} \times \vec{v} = \langle -2, 0, 2\rangle \times \langle 3, 4, -1\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 2 \\ 3 & 4 & -1 \end{vmatrix} = \langle -8, 4, -8\rangle$$

$$d = \frac{|\langle -8, 4, -8\rangle|}{|\langle 3, 4, -1\rangle|} = \frac{\sqrt{8^2 + 4^2 + 8^2}}{\sqrt{3^2 + 4^2 + 1}}$$

WHERE DO LINES INTERSECT?

ex.

LINE 1: $x = t$, $y = -t + 2$, $z = t + 1$ PARAM t

LINE 2: $x = 2s + 2$, $y = s + 3$, $z = 5s + 6$ PARAM s

SET EQUAL!

$t = 2s + 2$

$-t + 2 = s + 3$

$t + 1 = 5s + 6$

$-(2s+2) + 2 = s + 3$

$-2s = s + 3$

$-3s = 3$

$s = -1$

$\downarrow$

$(0, 2, 1)$

2nd PARTICLE HERE AT TIME $s = -1$

$t = 2(-1) + 2$

$t = 0$

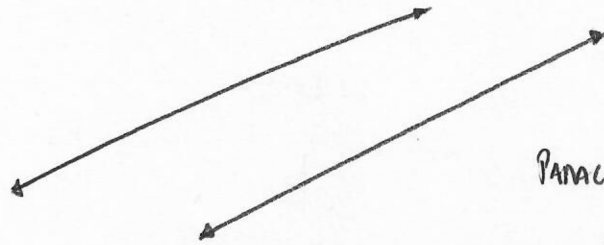
$\downarrow$

$(0, 2, 1)$

1st PARTICLE IS HERE AT TIME 0

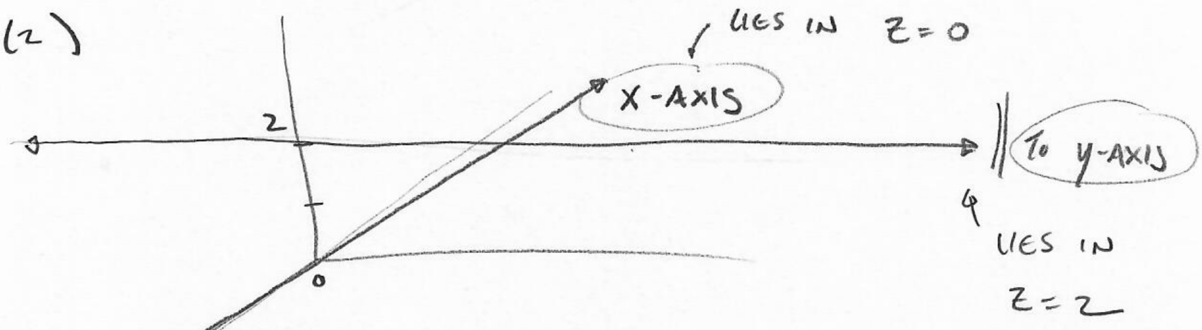
LINES THAT DO NOT INTERSECT :

(1)



PARALLEL LINES DO NOT INTERSECT (UNLESS THEY ARE IDENTICAL)

(2)



SKEW LINES

" l_1 & l_2 ARE <u>SKEW</u> "