§12.6 CYLINDERS & QUADRIC SURFACES

EQ MISSING ONE VARIABLE :

↓

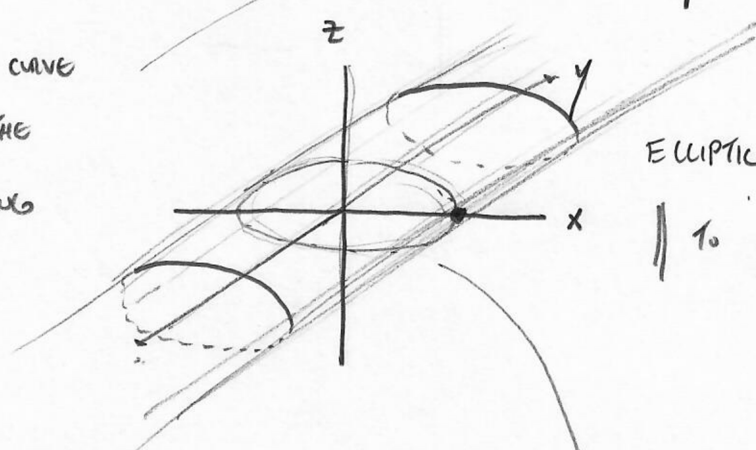
GRAPH IS A CYLINDER:

TAKE THE 2D CURVE

MOVE IT ∥ TO THE AXIS OF THE MISSING VARIABLE.

e.g. $\boxed{2z^2 + 3x^2 = 4}$

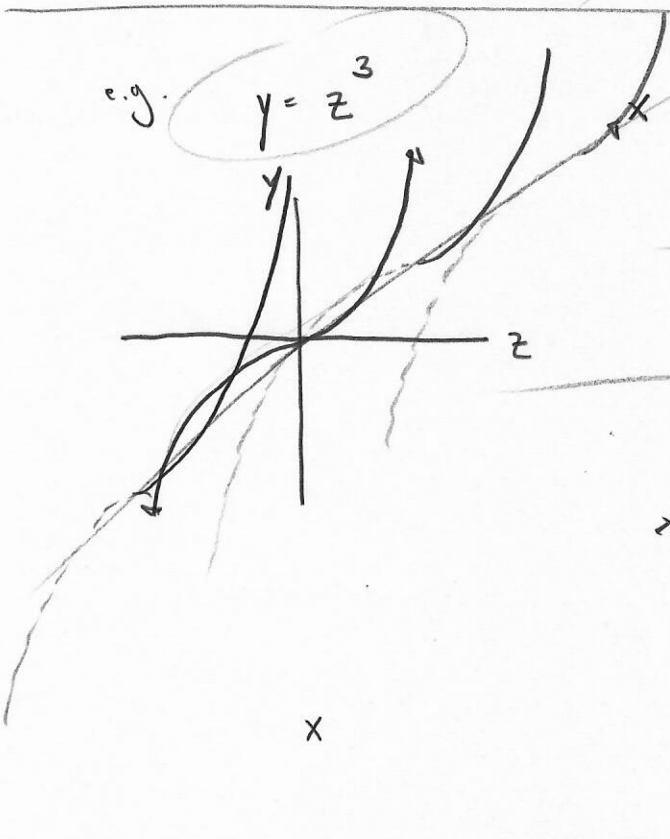
MISSING y

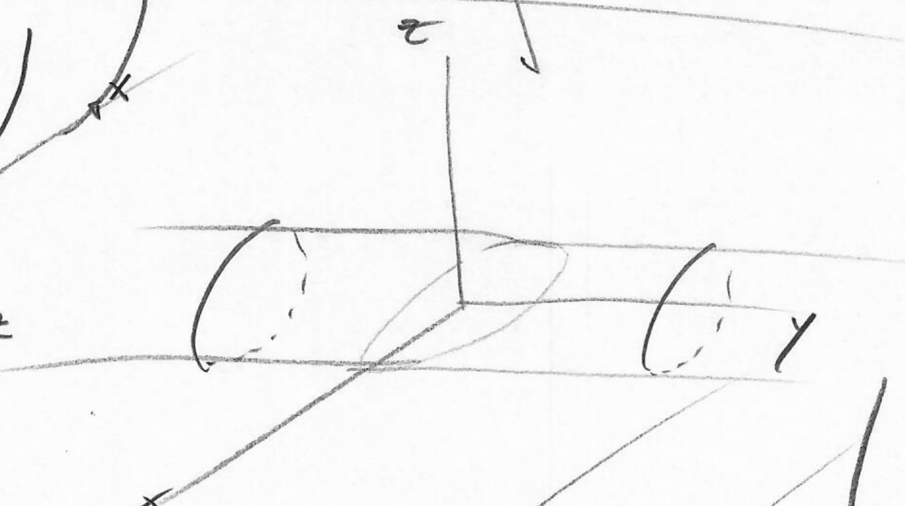


ELLIPTIC CYLINDER

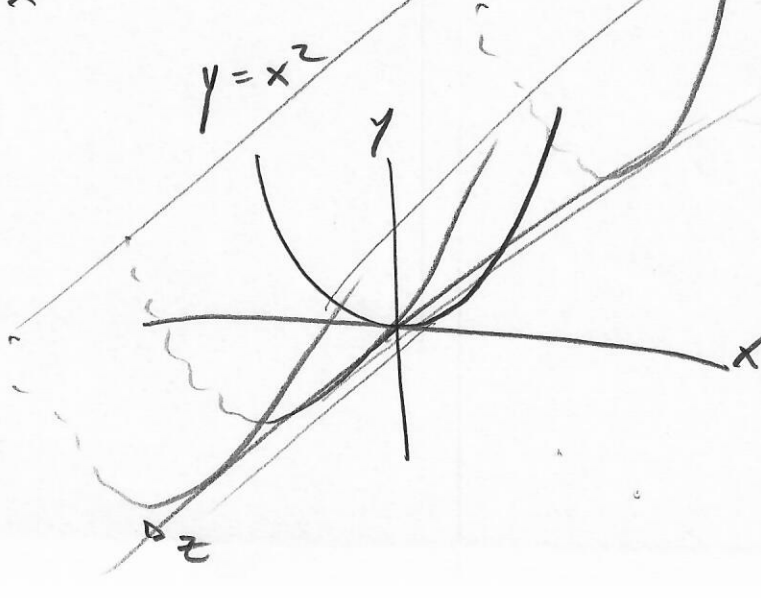
∥ to y-AXIS

e.g. $y = z^3$





$y = x^2$



QUADRIC SURFACE: GRAPH OF ANY 2^{nd} DEGREE EQ IN 3 VARIABLES.

$$\underbrace{Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz}_{2^{nd} \text{ DEGREE TERMS}} + Gx + Hy + Iz + J = 0$$

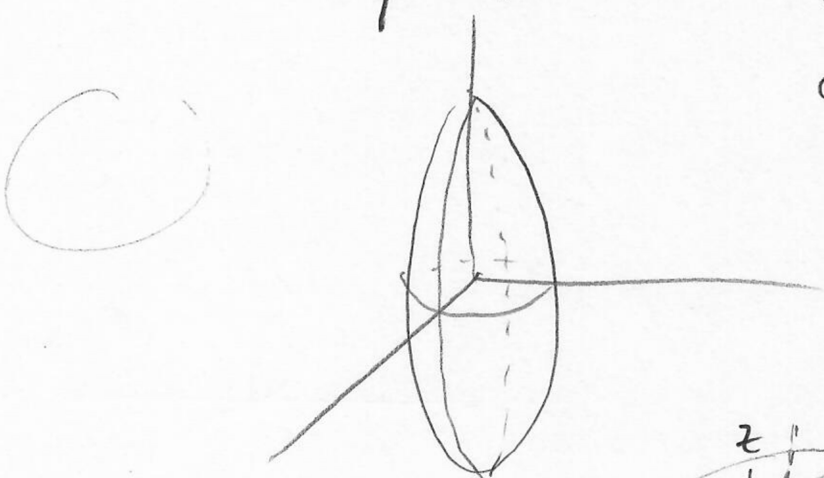
THESE CAN BE SIMPLIFIED ALGEBRAICALLY.

THEY ARE ALL TRANSFORMATIONS OF A FEW BASIC SHAPES.

ELLIPSOID $ax^2 + by^2 + cz^2 = r^2$

$a = b = c = 1 \Rightarrow$ SPHERE

$a, b, c > 0 \Rightarrow$ ELLIPSOID

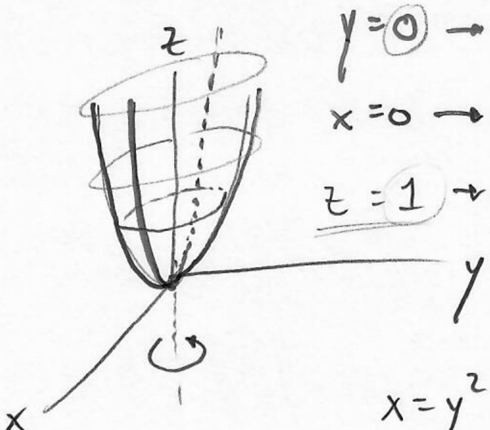


$y = 0 \rightarrow z = x^2$

$x = 0 \rightarrow z = y^2$

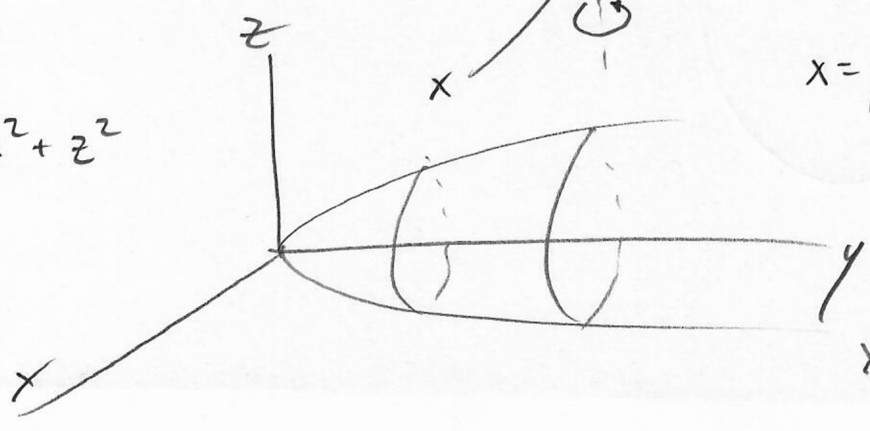
$z = 1 \rightarrow x^2 + y^2 = 1$

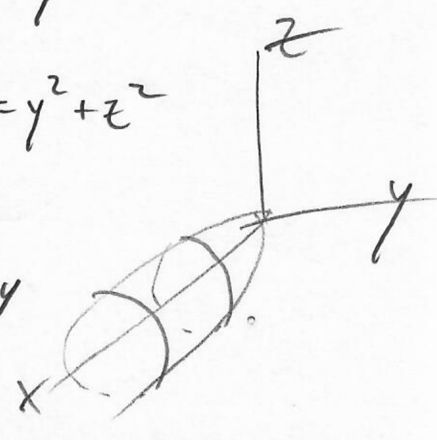
PARABOLOID: $z = x^2 + y^2$



$x = y^2 + z^2$

$y = x^2 + z^2$



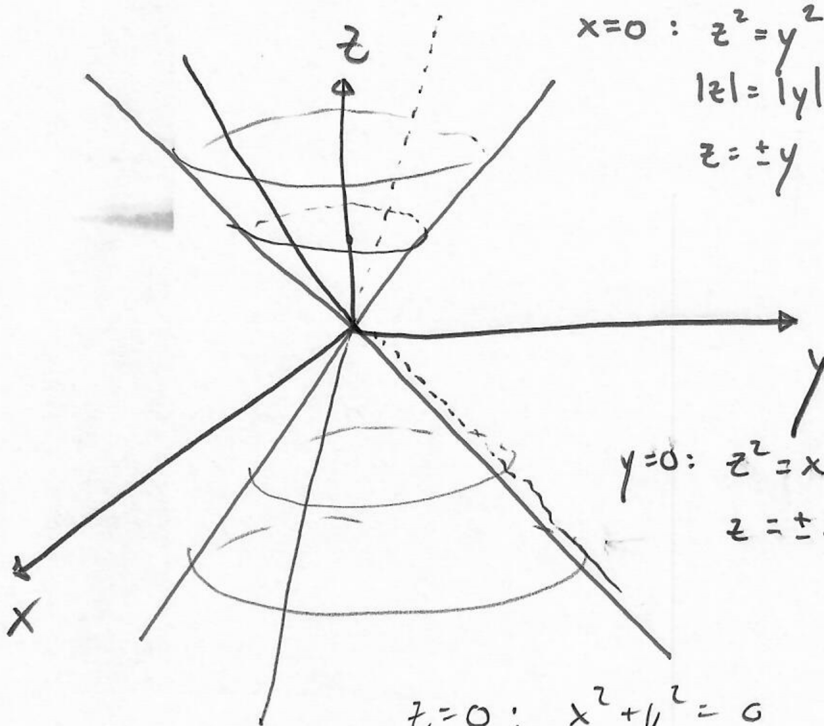


CONES:

$$z^2 = x^2 + y^2$$

$$y^2 = x^2 + z^2$$

$$x^2 = y^2 + z^2$$



$x=0$: $z^2 = y^2$

$|z| = |y|$

$z = \pm y$

$y=0$: $z^2 = x^2$

$z = \pm x$

$z=0$: $x^2 + y^2 = 0$

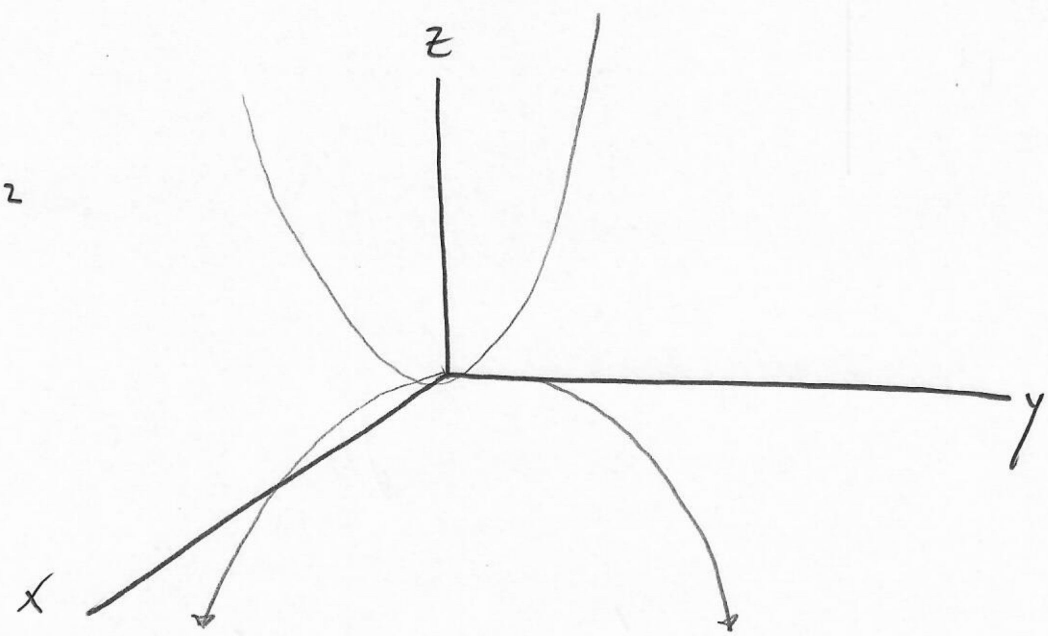
$x = y = 0$

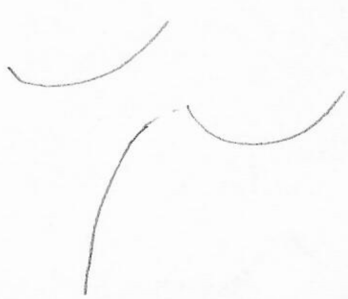
$\underline{z=1}$: $x^2 + y^2 = 1$

$\underline{z=c}$: $x^2 + y^2 = c^2$

HYPERBOLOID OF ONE SHEET:

$$z = x^2 - y^2$$





$\underline{x=0}$: $z = -y^2$

$y=0$: $z = x^2$

MOST IMPORTANT: CYLINDER **

PARABALOID.

ELLIPSOIDS.

§12.6

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§14.1 FUNCTIONS OF SEVERAL VARIABLES

AREA OF A RECTANGLE $A = lw$

$$A(l, w) = lw$$

VOLUME OF A BOX $V = lwh$, $V(l, w, h) = lwh$

DISTANCE FROM $P(x, y, z)$ TO $(3, 4, 5)$

$$d(x, y, z) = \sqrt{(x-3)^2 + (y-4)^2 + (z-5)^2}$$

Def: A FUNCTION OF 2 (3) VARIABLES IS A RULE THAT ASSIGNS TO EACH ORDERED PAIR (OR TRIPLET) OF REAL NUMBERS (x, y) $((x, y, z))$ IN A SET D A UNIQUE REAL NUMBER $f(x, y)$ $(f(x, y, z))$.

THE SET D IS CALLED THE DOMAIN, & THE SET OF VALUES f EQUALS IS CALLED THE RANGE.

ex. $f(x,y) = \sqrt{xy}$. FIND $f(-4,-36) = \sqrt{(-4)(-36)}$

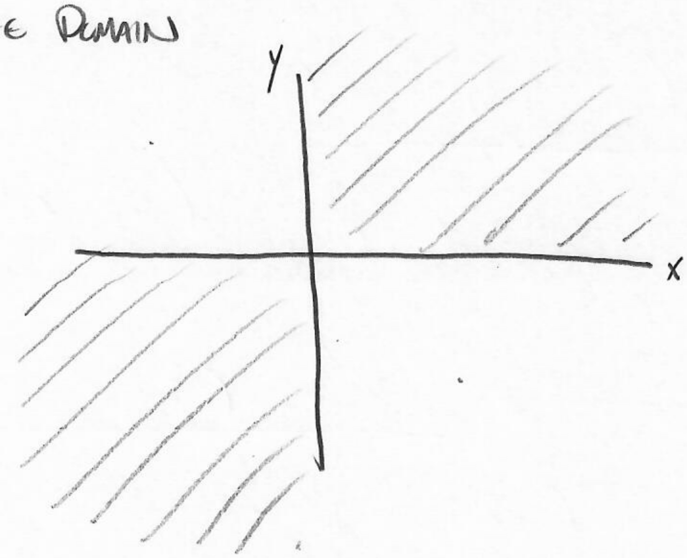
$= 12$

$\text{Dom}(f) = \{ (x,y) \mid f(x,y) \text{ IS DEFINED} \}$

$= \{ (x,y) \mid xy \geq 0 \}$

- $x = 0$, or $y = 0$
- or $x \geq 0$, $y \geq 0$ BOTH POS.
- or $x \leq 0$, $y \leq 0$ BOTH NEG.

SKETCH THE DOMAIN

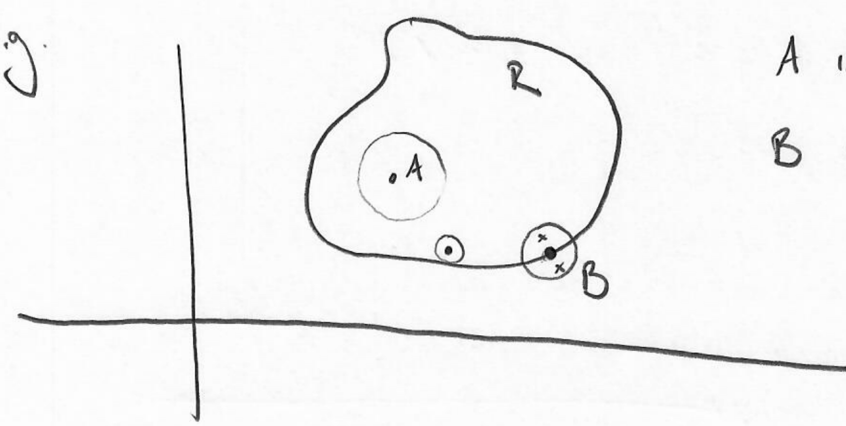


Def: A SUBSET OF THE XY-PLANE IS CALLED A <u>REGION</u> R,

(THE DOMAIN OF A FUNCTION OF 2 VAR'S IS A REGION).

A POINT (x_0, y_0) IS CALLED AN <u>INTERIOR POINT OF</u> R IF IT IS THE CENTER OF A DISK (OF ANY RADIUS) CONTAINED IN R. IT IS CALLED A <u>BOUNDARY POINT</u> IF EVERY DISK CENTERED AT THIS POINT CONTAINS POINTS BOTH POINTS IN R & POINTS NOT IN R.

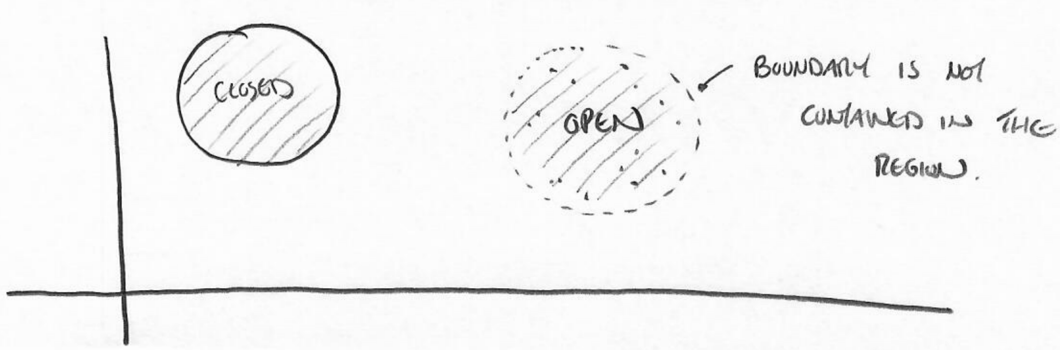
e.g.



A INTERIOR PNT OF R

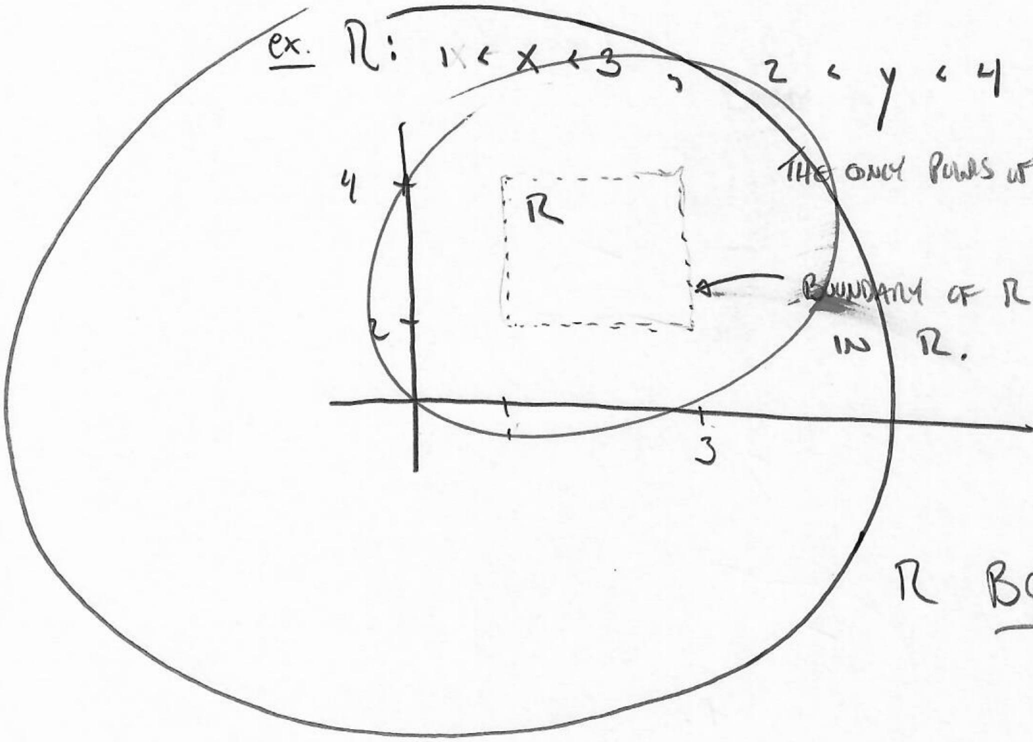
B BOUNDARY PNT OF R

A REGION R IS CALLED <u>OPEN</u> IF EVERY POINT IN R IS AN INTERIOR POINT, AND R IS CALLED <u>CLOSED</u> IF IT CONTAINS ALL OF ITS BOUNDARY POINTS.



A region R is called **Bounded** if it is contained in some disk of finite radius. Otherwise it is unbounded.

ex. $R: 1 < x < 3, \quad 2 < y < 4$

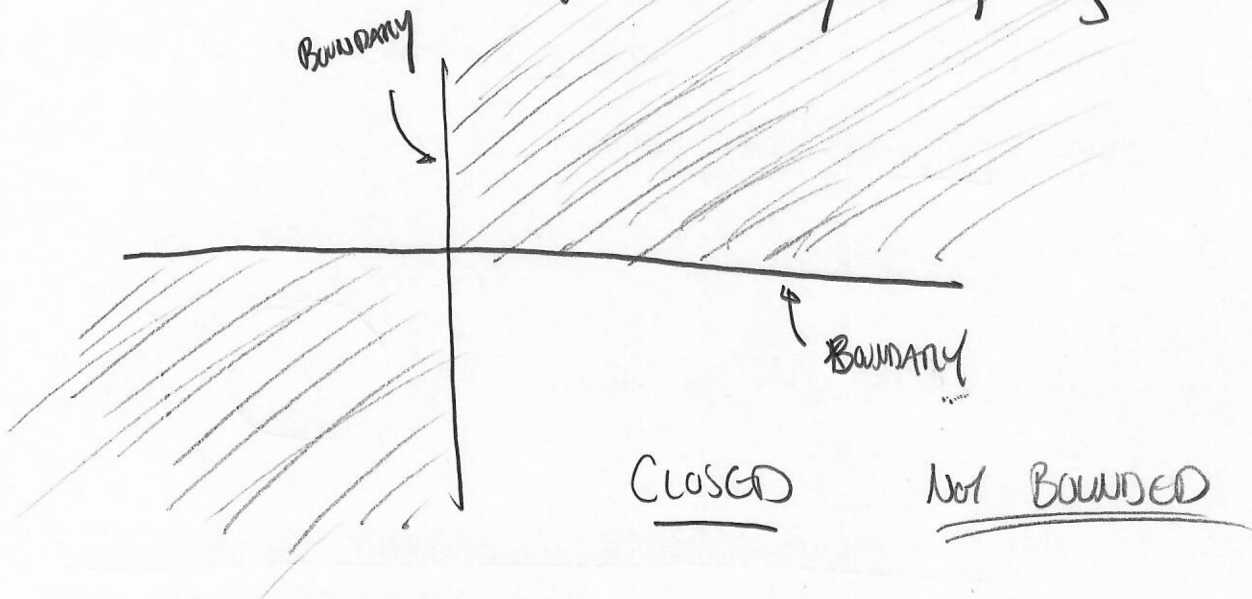


The only points of R are interior $\Rightarrow$ OPEN

Boundary of R is not contained in R.

R BOUNDED ✓

ex. $R = \{(x,y) : (x \geq 0 \text{ AND } y \geq 0) \text{ or } (x \leq 0 \text{ AND } y \leq 0)\}$



CLOSED NOT BOUNDED

ex. FIND DOMAIN & RANGE & FIND BOUNDARY POINTS OF THE DOMAIN.

IS DOMAIN $\underbrace{\text{OPEN ? CLOSED ?}}$ BOUNDED ?

(NOTE: REGIONS CAN BE BOTH OR NEITHER.)

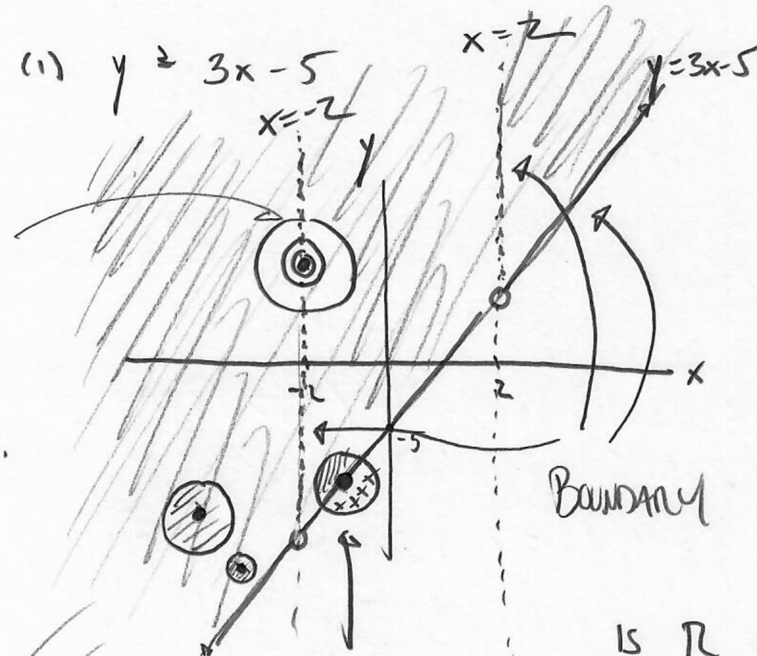
(a) $f(x,y) = \dfrac{\sqrt{y-3x+5}}{x^2-4}$

$$\text{Dom}(f) = \{ (x,y) \mid \overset{(1)}{y-3x+5 \geq 0}, \ \overset{(2)}{x^2-4 \neq 0} \}$$

(1) $y \geq 3x-5$

(2) $x^2 \neq 4$

$x \neq \pm 2$



R DOES NOT CONTAIN ITS BOUNDARY

$\Rightarrow$ NOT CLOSED.

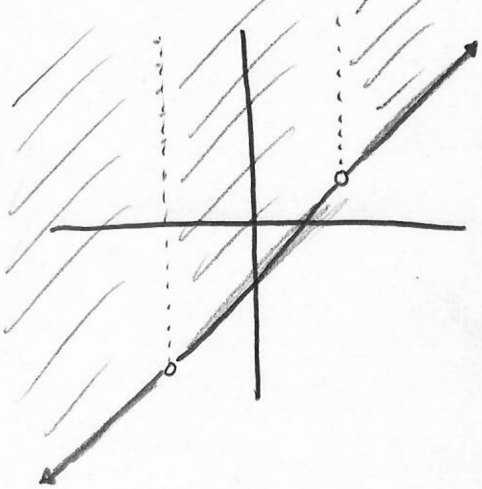
IS IN R

BUT NOT INTERIOR

$\Rightarrow$ R IS NOT OPEN.

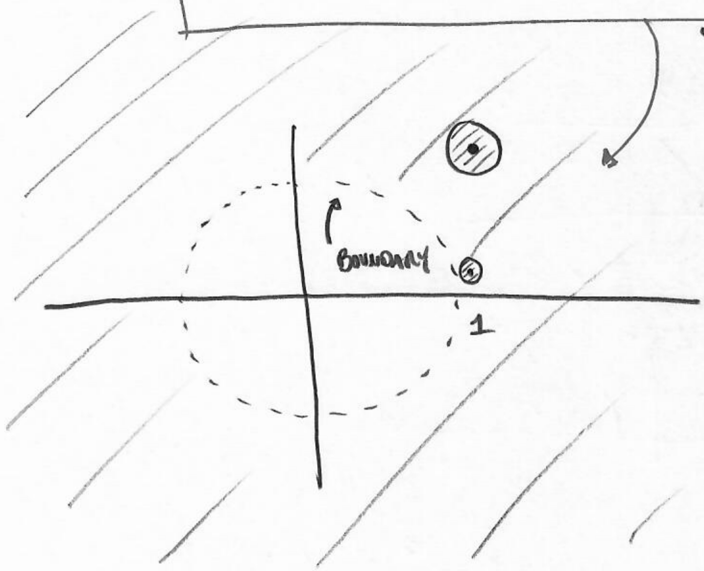
IS R OPEN?

EVERY PT IN R IS INTERIOR PNT



(b) $f(x,y) = \ln(x^2+y^2-1)$

$$\boxed{\text{Dom}(f) = \{ (x,y) \mid \underbrace{x^2+y^2-1>0}_{x^2+y^2>1} \}}$$



BOUNDARY NOT CONTAINED IN DOM

$\Rightarrow$ DOMAIN IS NOT CLOSED.

EVERY POINT IN DOM IS INTERIOR

$\Rightarrow$ DOMAIN IS OPEN.

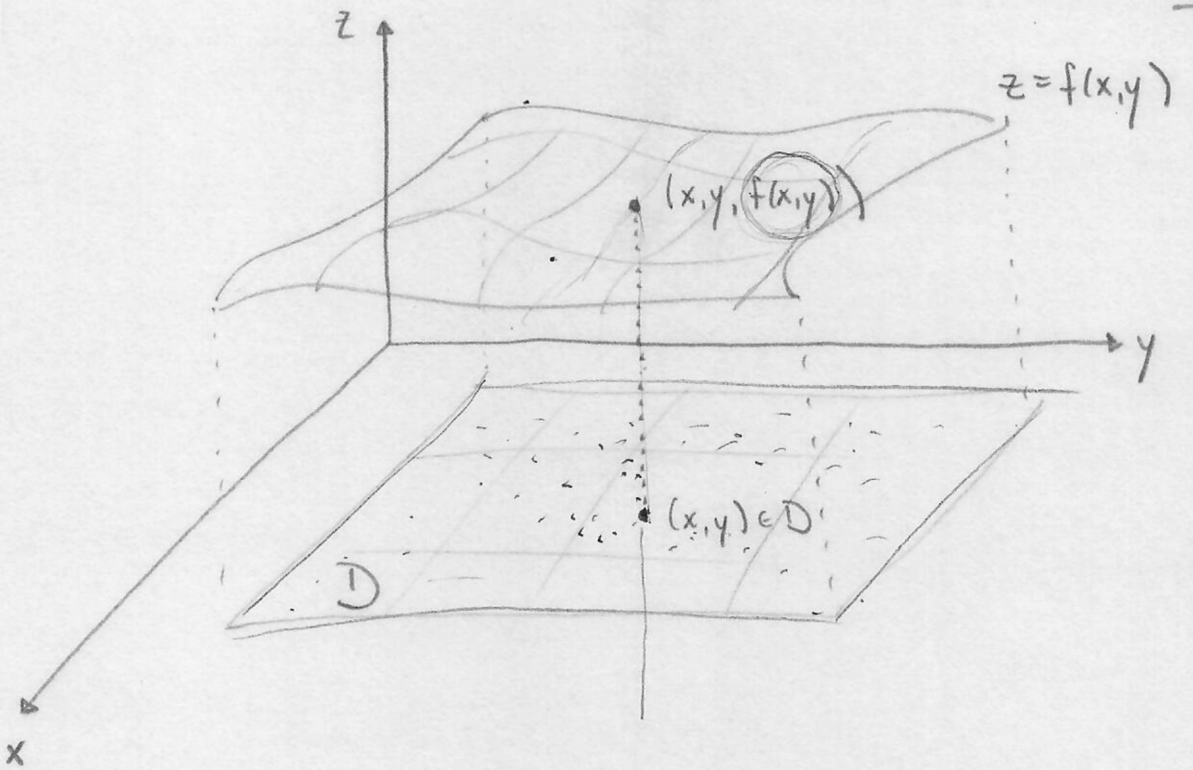
BOUNDED? No. UNBOUNDED.

GRAPHS : 2 VAR.

Def: GIVEN FUNCTION f OF 2 VARIABLES WITH DOMAIN D,

ITS GRAPH IS THE SET OF ALL POINTS (x, y, z) IN $\mathbb{R}^3$

S.T. $\boxed{z = f(x,y)}$ & $(x,y) \in D$. (ALSO CALLED A SURFACE)



Def: GIVEN A CONSTANT c, THE SET OF POINTS IN D S.T.

$f(x,y) = c$ IS CALLED A LEVEL CURVE OF f.

ex. CONSIDER $f(x,y) = x^2 + 2y^2$

SKETCH LEVEL CURVES FOR $c = 0, 1, 2,$ ETC.

SKETCH GRAPH / SURFACE $z = f(x,y)$.

ex. FIND EQ OF LEVEL CURVE OF $f(x,y) = \dfrac{2y-1}{3x+4y-2}$

THROUGH $(1,1)$.

ex. CONSIDER $f(x,y) = x^2 + 2y^2$

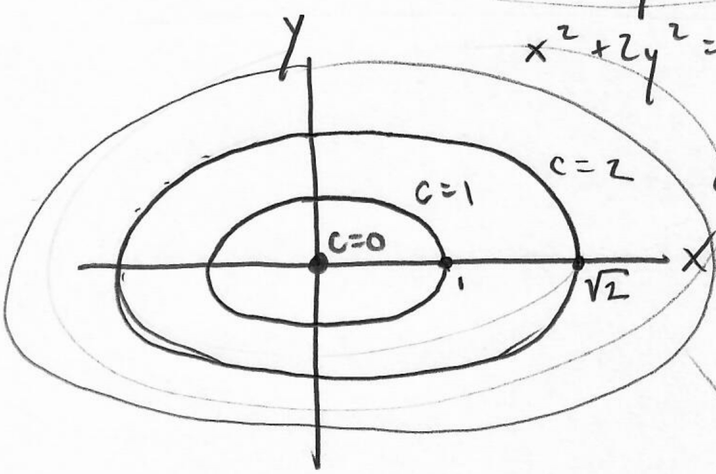
SKETCH $\underbrace{\text{LEVEL CURVES}}$ FOR $c = 0,\ c = 1,\ c = 2,\ \ldots$

$f(x,y) = c$

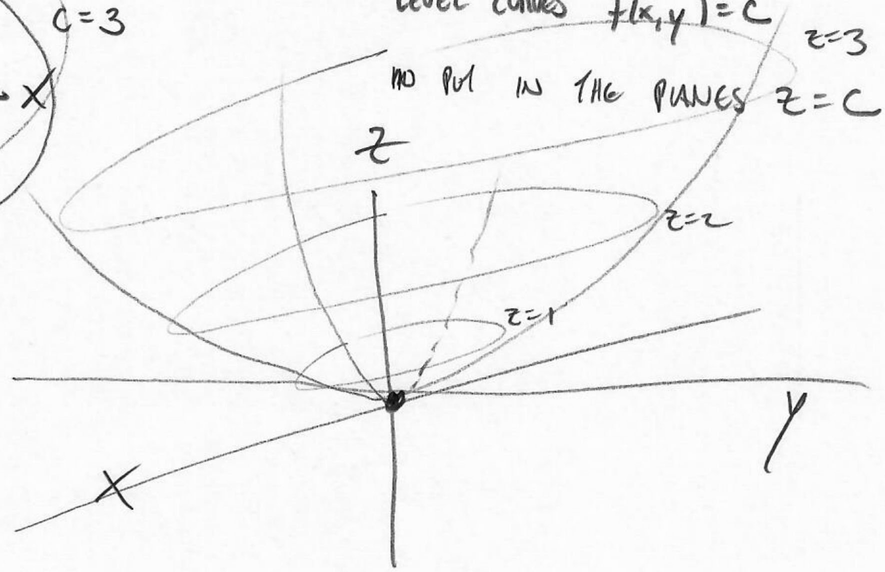
$x^2 + 2y^2 = 0 \rightarrow (0,0)$

$x^2 + 2y^2 = 1$

$x^2 + 2y^2 = 2$



GRAPH $z = f(x,y)$ HAS LEVEL CURVES $f(x,y) = c$ AND PUT IN THE PLANES $z = c$



NOTE: EVERY POINT IN THE DOMAIN LIES ON ONE (AND ONLY ONE) LEVEL CURVE.

GIVEN $(a,b) \in \text{Dom}(f)$

SET $c = f(a,b)$.

THEN (a,b) LIES ON LEVEL CURVE $f(x,y) = c$.

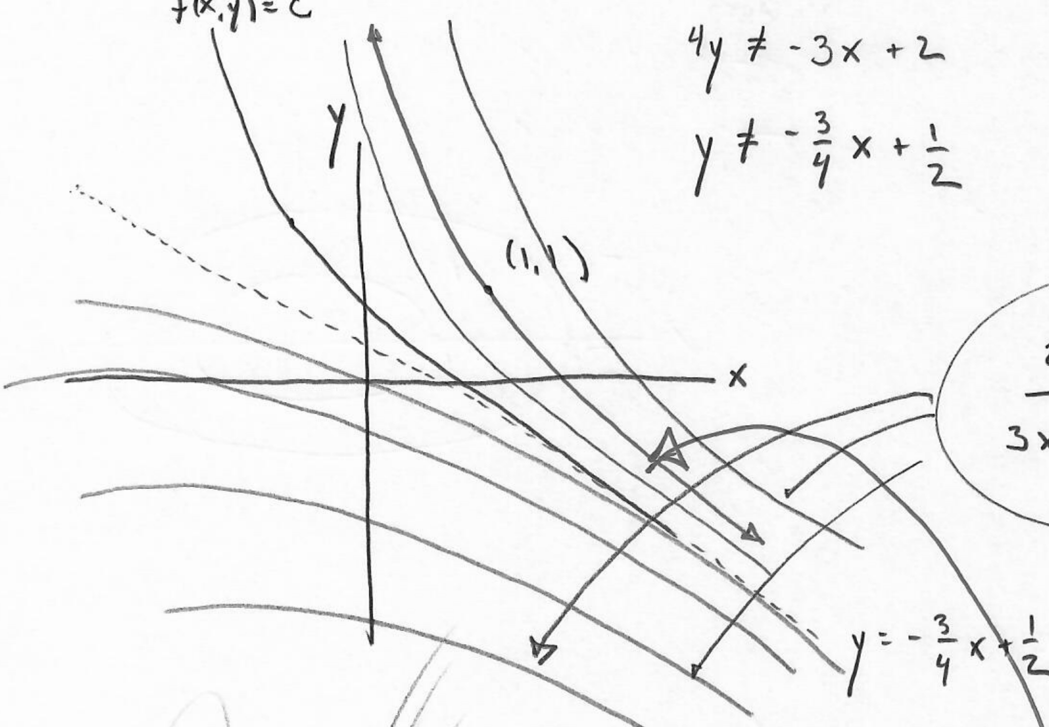
FIND THE EQ OF THE LEVEL CURVE OF $f(x,y) = \dfrac{2y-1}{3x+4y-2}$

THROUGH THE POINT $(1,1)$.

$\text{Dom}(f) = \{(x,y) \mid 3x+4y-2 \neq 0\}$

$4y \neq -3x+2$

$y \neq -\frac{3}{4}x + \frac{1}{2}$

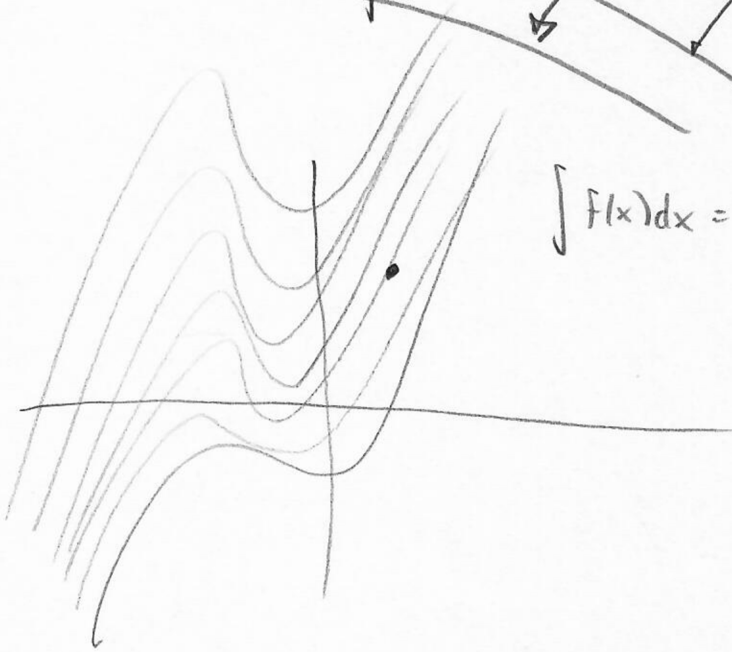


THE LEVEL CURVES FILL THE DOMAIN.

$$\frac{2y-1}{3x+4y-2} = c$$

$x=1, \; y=1$

$$\frac{2(1)-(1)}{3(1)+4(1)-2} = \frac{1}{5} = c$$



$\int f(x)\,dx = F(x) + C$

$$\boxed{\frac{2y-1}{3x+4y-2} = \frac{1}{5}}$$

IN 3D, WE HAVE LEVEL SURFACES

Def: GIVEN A CONSTANT c, THE SET OF ALL POINTS $(x,y,z) \in Dom(f)$ S.T. $f(x,y,z) = c$ IS CALLED A LEVEL SURFACE OF f.

ex. FIND EQ OF LEVEL SURFACE FOR

$$f(x,y,z) = \ln(x^2+y^2+z^2)$$

PASSING THROUGH $(-1,2,1)$

$\ln(x^2+y^2+z^2) = c$ SATISFIED BY

$x=-1, y=2, z=1$

$$\ln((-1)^2 + (2)^2 + 1^2) = \ln(6) = c$$

$$\boxed{\ln(x^2+y^2+z^2) = \ln 6}$$

$$x^2+y^2+z^2 = 6$$