§14.3 $\boxed{w = (7x^2)(\tan(6x^3y))}$

MIXED 2^{nd} ORDER PARTIAL DERIVATIVES.

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} w\right) = \frac{\partial^2 w}{\partial y\, \partial x} = \frac{\partial^2 w}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} w\right)$$

$$w_{xy} = w_{yx}$$

SAME!

$$\frac{\partial w}{\partial x} = w_x = 14x \tan(6x^3y) + \underbrace{7x^2 \sec^2(6x^3y)\, 18x^2y}_{(126x^4y)(\sec^2(6x^3y))}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}\right) = \frac{\partial^2 w}{\partial y \partial x} = w_{xy} = 14x \sec^2(6x^3y)\, 6x^3$$

$$+ 126x^4 \sec^2(6x^3y) +$$

$$+ 126x^4y \cdot 2\sec(6x^3y)\sec(6x^3y)\tan(6x^3y) \cdot 6x^3$$

w_{yx}

$$\left[6\left(x-\tfrac{5}{6}\right)\right]^{2n+1} \to \frac{6^{2n+1}}{n^{3/2}}\boxed{\left(x-\tfrac{5}{6}\right)^{2n+1}}$$

$$\sum_{n=1}^{\infty} \frac{(6x-5)^{2n+1}}{n^{3/2}} \qquad C_n$$

SERIES CONVERGES ABS. WHEN $\rho < 1$, DIV. WHEN $\rho > 1$

INCONCLUSIVE WHEN $\rho = 1$.

RATIO TEST: $$\rho = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(6x-5)^{\overset{2n+3}{2(n+1)+1}}}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{(6x-5)^{2n+1}}\right|$$

$$= \lim_{n\to\infty} \underbrace{\left|\left(\frac{n}{n+1}\right)^{3/2}\right|}_{1}\underbrace{\left|(6x-5)^2\right|}$$

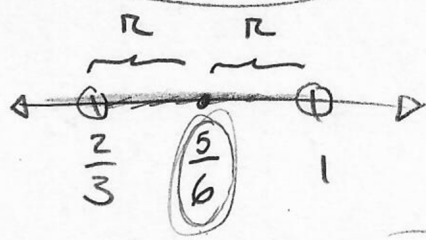
$$0 \le \rho = (6x-5)^2 < 1$$

RADIUS OF CONV. $R = \frac{1}{6}$

$$-1 < 6x-5 < 1$$

$$4 < 6x < 6$$

$$\frac{2}{3} < x < 1$$



$$\sum_{n=0}^{\infty} c_n (x-a)^n$$

$$x = \frac{2}{3}: \quad \sum_{n=1}^{\infty}\left|\frac{(-1)^{2n+1}}{n^{3/2}}\right| = \cancel{(-1)}\sum_{n=1}^{\infty}\frac{\overset{1}{\cancel{(-1)^{2n}}}}{n^{3/2}}$$

$p = 3/2 > 1$

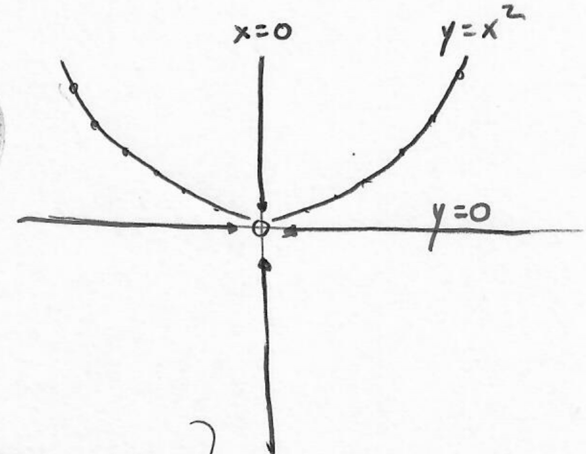
CONVERGES ABS. AT $x = \frac{2}{3}$

$x = 1$: $\sum_{n=1}^{\infty} \left| \frac{1}{n^{3/2}} \right|$ CONV. ABS.

SERIES CONV. ABS. $\frac{2}{3} \leq x \leq 1$

$\left[\frac{2}{3}, 1 \right]$

$$\lim_{(x,y)\to(0,0)} h(x,y) = \frac{x^2 y}{x^4 + y^2}$$



$$\lim_{\substack{y=0\\x\to 0}} h(x,0) = \lim_{x\to 0} \frac{0}{x^4} = 0$$

$$\lim_{y\to 0} h(0,y) = \lim_{y\to 0} \frac{0}{y^2} = 0$$

SET $y = x^2 \longrightarrow h(x, x^2) = \dfrac{x^2(x^2)}{x^4 + (x^2)^2} = \dfrac{x^4}{2x^4} = \dfrac{1}{2}$

$$\lim_{\substack{y=x^2\\x\to 0}} \frac{x^4}{2x^4} = \frac{1}{2}$$

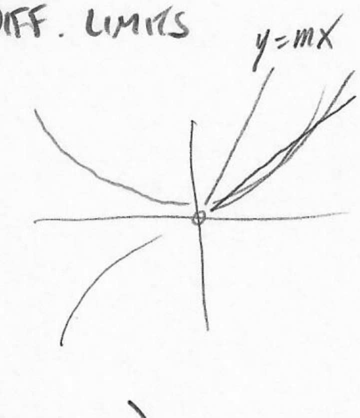
DIFFERENT LIMITS ALONG DIFFERENT PATHS THROUGH $(0,0) \implies$ LIMIT D.N.E.

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

$y=0$, $x\to 0$ ✓
$x=0$, $y\to 0$ ✓
$y=x$, $x\to 0$
$y=x^2$, $x\to 0$

PATHS TO TRY WHEN LOOKING FOR DIFF. LIMITS ALONG DIFF. PATHS:

- x-AXIS ($y=0$, $x\to 0$)
- y-AXIS ($x=0$, $y\to 0$)
- $y=mx$, $x\to 0$
- $y=x^n$, $x=y^n$ $(n=2,3,\dots)$



$$\lim_{(x,y)\to(0,0)} \frac{y+\sin x}{x+\sin y}$$

X-AXIS: $y=0$, $x\to 0$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Y-AXIS: $x=0$, $y\to 0$

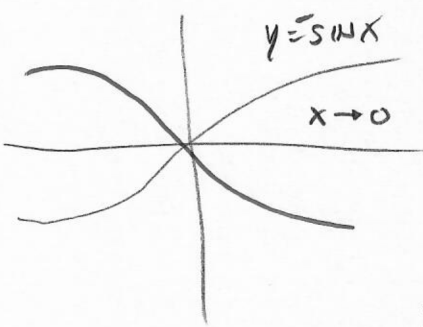
$$\lim_{y\to 0} \frac{y}{\sin y} = 1$$

$y=x$: $$\lim_{x\to 0} \frac{x+\sin x}{x+\sin x} = 1$$

$y=mx$, $x\to 0$ $$\lim_{x\to 0} \frac{mx+\sin x}{x+\sin(mx)} \qquad \frac{m+\cos x}{1+m\cos x}$$

$y=2x$, $x\to 0$ $$\lim_{x\to 0} \frac{2x+\sin x}{x+\sin(2x)}$$

$\uparrow$
$2\sin x\cos x$



$$\lim_{x\to 0} \frac{-\sin x+\sin x}{x+\sin(-\sin x)} = \lim_{x\to 0} \frac{\overset{0}{\cancel{2\sin x}}}{x+\sin(\sin x)} ; \frac{0}{0}$$

$$\xrightarrow{\text{L'Hô}} \lim_{x\to 0} \frac{2\cos x}{1+\cos(\sin x)\cos x} = \frac{2}{1+1} = 1$$

$$\lim_{(x,y)\to(0,0)} \frac{-5e^{-4y}\sin(-5x)}{-5x}$$

Let $t = -5x$

Then $t \to 0$ as $x \to 0$

$$\lim_{(x,y)\to(0,0)} \underbrace{\left(-5e^{-4y}\right)}_{-5} \underbrace{\left(\frac{\sin(-5x)}{-5x}\right)}_{1}$$

$$\lim_{t\to 0} \frac{\sin t}{t} = 1$$

$$= -5$$

$f(x) = (1-x+x^2)e^x$; $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

$$(1 - x + x^2)\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots\right)$$

$$= 1 \underbrace{+x - x}_{0} \underbrace{+ x^2 + \frac{1}{2}x^2 - x^2}$$

$$= 1 + \frac{1}{2}x^2 + \underbrace{\frac{1}{6}x^3 - \frac{1}{2}x^3 + x^3}$$

$$= \boxed{1 + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \dots}$$

$f(x) = x^4 \cos(3x)$ FIND FIRST 4 TERMS OF MACLAURIN SERIES

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n} = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots$$

$$f(x) = x^4\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{6!}x^6 + \dots\right)$$

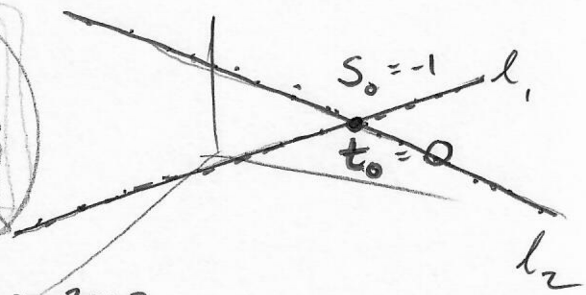
$$f(x) = x^4 - \frac{1}{2}x^6 + \frac{1}{24}x^8 - \frac{1}{6!}x^{10} + \dots$$

$$\cos(3x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}(3x)^{2n} = \sum_{n=0}^{\infty} \frac{(-1)^n 9^n}{n!} x^{2n}$$

$$f(x) = x^4 \cdot 9^0 - \frac{1}{2}9^1 x^6 + \frac{1}{24}9^2 x^8 - \frac{1}{6!}9^3 x^{10}$$

l_1: $x = 3t + 3$ $\qquad y = 2t + \cancel{3}$ 4 $\qquad z = 3t + \cancel{5}$ 4

$x = s + 4$ $\qquad y = 2s + 5$ $\qquad z = -2s + 3$



LOOKING FOR t_0 & s_0 $\qquad 3t+5 = -2s+3$

SUCH THAT THEY PRODUCE THE SAME x, y, t VALUES.

$3t + 3 = s + 4$ $\qquad 2t + 3 = 2s + 5$

~~2~~ 3 EQ
2 UNKNOWNS
SOLVE!

$3t - 1 = s$ $\qquad 2t + 3 = 2(3t - 1) + 5$

$2t + \cancel{3} = 6t \cancel{-2} \cancel{+5}$

$3(0) - 1 = s$ $\qquad 0 = 4t$

$\boxed{s = -1}$ $\qquad \boxed{t = 0}$

$\boxed{x = 3,\ y = 3,\ 5}$ $\qquad x = 3 \quad y = 3 \quad z = 5$

$(3, 3, 5)$

CENTER $(1, 0, -7)$

SPHERE $(x-1)^2 + y^2 + (z+7)^2 = 16$

FIND POINT ON SPHERE CLOSEST TO

$3x - 11y + 6z = 5$

(a) xy-PLANE $(1, 0, -3)$

(b) THE POINT $(8, 0, -7)$ $(5, 0, -1)$