§13.1 VECTOR FIELDS

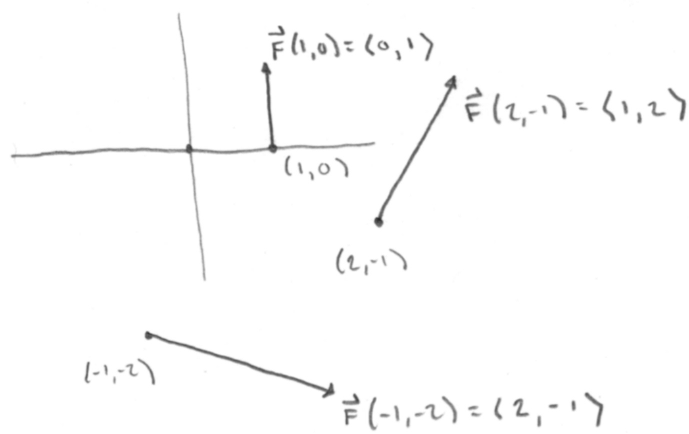
DEF: LET $D \subset \mathbb{R}^2$. A <u>VECTOR FIELD ON $\mathbb{R}^2$</u> IS A FUNCTION $\vec{F}$ THAT ASSIGNS TO EACH POINT $(x, y) \in D$ A 2 DIMENSIONAL VECTOR $\vec{F}(x,y)$.

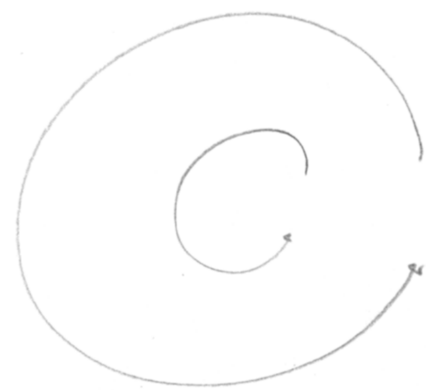
VELOCITY FIELDS

WEATHER MAPS WIND

WATER CURRENT

ex. SKETCH VECTOR FIELD $\vec{F}(x,y) = \langle -y, x \rangle$





DEF: LET $E \subset \mathbb{R}^3$. A <u>VECTOR FIELD ON $\mathbb{R}^3$</u> IS A FUNCTION $\vec{F}$ THAT ASSIGNS TO EACH POINT $(x, y, z) \in E$ A 3D VECTOR $\vec{F}(x,y,z)$.

FORCE FIELDS

GRAVITATIONAL FORCE FIE

ex. SKETCH VECTOR FIELD $\vec{F}(x,y,z) = \langle -x, -y, -z \rangle$

REMARK: HERE, OUR COMPONENT FUNCTIONS ARE REAL-VALUED FUNCTIONS OF n VARIABLES ($n = 2$ or 3).

$$\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$$

$$\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$$

IDENTIFYING POINT (x, y, z) WITH CORRESPONDING POSITION VECTOR $\vec{x} = \langle x, y, z \rangle$, WE CAN THINK OF A VECTOR FIELD AS A FUNCTION ASSIGNS VECTORS TO VECTORS.

ex. SHOW THAT EACH VECTOR OF $\vec{F}(x,y) = \langle -y, x \rangle$ IS TANGENT TO A CIRCLE CENTERED AT ORIGIN.

POSITION VECTOR $\vec{x} \perp \vec{F}(\vec{x})$

$$\langle x, y \rangle \cdot \langle -y, x \rangle = -xy + xy = 0 \quad \checkmark$$

FURTHERMORE $|\vec{F}(\vec{x})| = |\vec{x}|$

$$\sqrt{(-y)^2 + x^2} = \sqrt{x^2 + y^2} \quad \checkmark$$

$\boxed{\text{LOOK AT SOME COMPUTER SKETCHES}}$

GRADIENT FIELDS

IF f IS A SCALAR FUNCTION OF n VARIABLES, THEN ∇f IS AN n-DIM VECTOR FIELD.

$$\nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle$$

$$\nabla f(x,y,z) = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z) \rangle$$

ex. FIND GRADIENT VECTOR FIELD FOR $f(x,y) = \sqrt{x^2 + y^2}$

($\perp$ TO CONTOUR LINES)

Def: A VECTOR FIELD $\vec{F}$ IS CALLED CONSERVATIVE IF IT IS THE GRADIENT FIELD FOR SOME SCALAR FUNCTION, $\vec{F} = \nabla f$ FOR SOME f. IN THIS CASE, f IS CALLED A POTANTIAL FUNCTION FOR $\vec{F}$.