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§13.1 Vector Fields

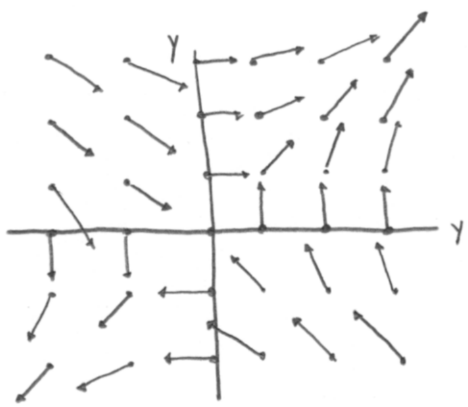
<u>5.</u> $\vec{F}(x,y) = \dfrac{1}{\sqrt{x^2+y^2}} \langle y , x \rangle$

Note that domain is $\mathbb{R}^2 \setminus \{(0,0)\}$

and all vectors are unit length.

Also $(x,y) \longmapsto (y,x)$ is reflection across

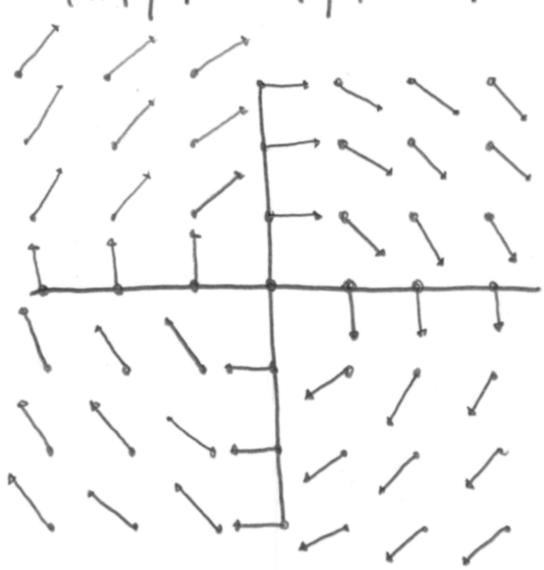
diagonal $y = x$



<u>6.</u> $\vec{F}(x,y) = \dfrac{1}{\sqrt{x^2+y^2}} \langle y , -x \rangle$

Domain : $\mathbb{R}^2 \setminus \{(0,0)\}$, unit length

$(x,y) \longmapsto (y,-x)$ is 90° rotation clockwise



9. $\vec{F}(x,y,z) = \langle 0, 0, x \rangle$

DOMAIN IS $\mathbb{R}^2$

$\vec{F}\Big|_{yz\text{-PLANE}} = \vec{0}$

FOR $x > 0$, VECTOR POINT "UP"
FOR $x < 0$, VECTORS POINT "DOWN"
} LENGTH = DISTANCE TO YZ-PLANE (i.e. $|x|$)

HARD TO DRAW ... SEE COMPUTER GRAPHIC BELOW.

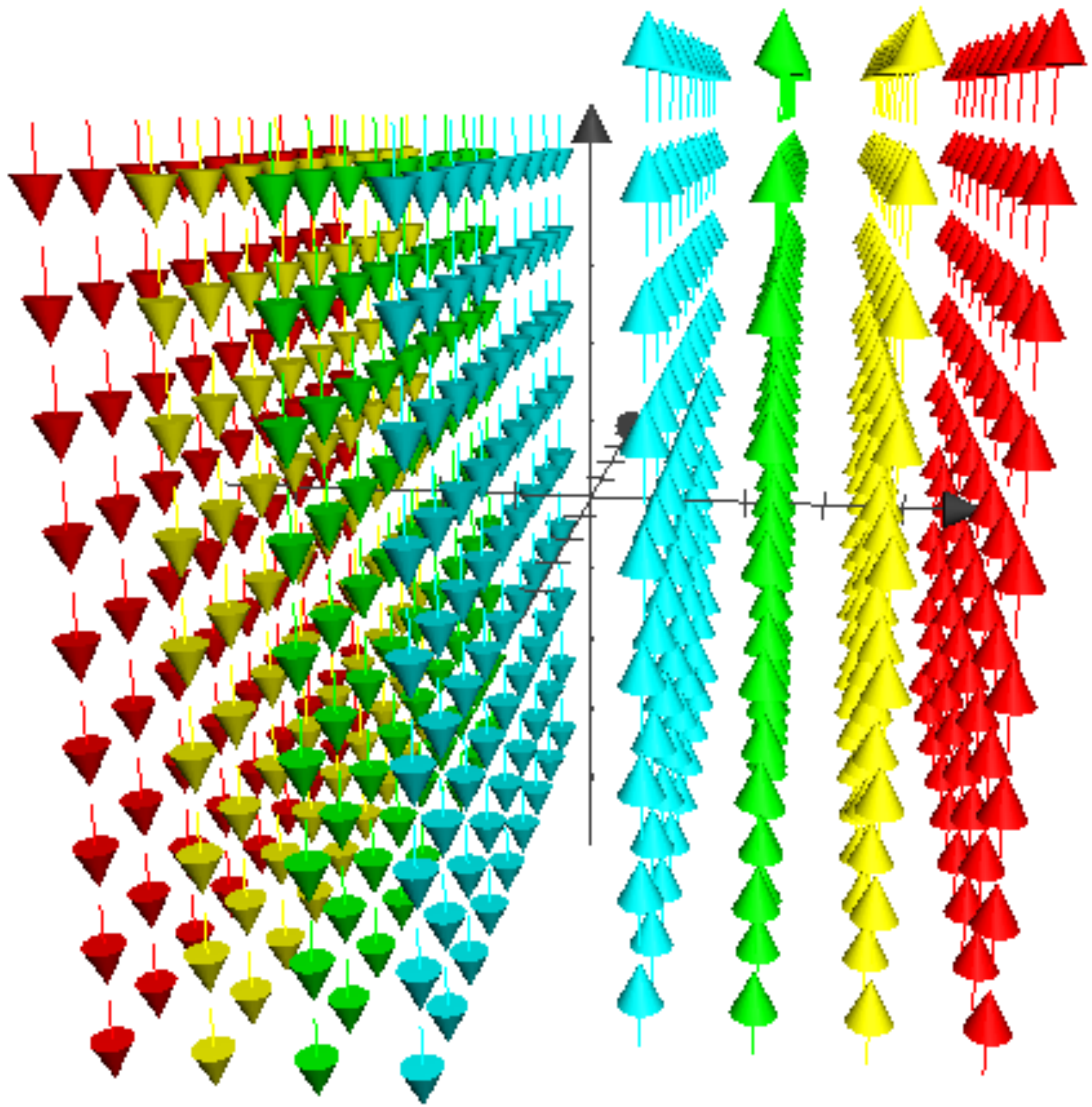
11. IV VECTOR ABOVE X-AXIS POINT DOWN
" BELOW " " UP

12. III VECTORS ABOVE X-AXIS POINT RIGHT
" BELOW " " DOWN

AND VECTORS ALONG DIAGONAL $y = x$ ARE HORIZONTAL

13. I VECTOR DO NOT DEPEND ON x
(SAME VECTOR ACROSS LINES $y = c$)

14. II PERIODIC ALONG VERTICAL LINES $x = c$



15. IV CONSTANT VECTOR FIELD

16. I VECTORS POINT MORE & MORE AWAY FROM XY-PLANE AS $|z|$ INCREASES.

17. III VECTORS VERTICLE ALONG Z-AXIS $(x=y=0)$, ALL HAVE SAME Z-COMPONENT

18. II ALL VECTOR POINT AWAY FROM ORIGIN.

21. $f(x,y) = xe^{xy}$

$$\nabla f(x,y) = \langle f_x, f_y \rangle$$

$$\boxed{\nabla f(x,y) = \langle e^{xy} + xye^{xy}, x^2 e^{xy} \rangle}$$

23. $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

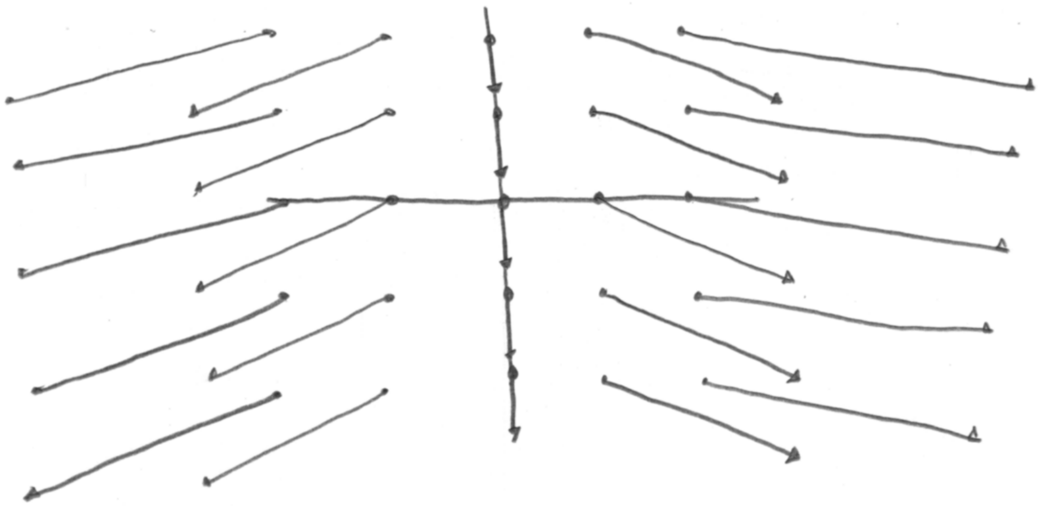
$$\nabla f(x,y,z) = \langle f_x, f_y, f_z \rangle$$

$$= \left\langle \frac{2x}{2\sqrt{x^2+y^2+z^2}}, \frac{2y}{2\sqrt{x^2+y^2+z^2}}, \frac{2z}{2\sqrt{x^2+y^2+z^2}} \right\rangle$$

$$\boxed{\nabla f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \langle x, y, z \rangle}$$

25. $f(x,y) = x^2 - y$

$\nabla f = \langle f_x , f_y \rangle = \langle 2x , -1 \rangle$



29. WHEN THE PARTICLE IS AT $(2,1)$, IT EXPERIENCES VELOCITY $\vec{V}(2,1) = \langle 2^2 , 2 + 1^2 \rangle = \langle 4, 3 \rangle$

SO FOR SMALL Δt, ITS POSITION IS APPROXIMATELY

$$\vec{r}(\Delta t) = \langle 2, 1 \rangle + \Delta t \langle 4, 3 \rangle$$

SET $\Delta t = 3.01 - 3 = 0.01$

POSITION $\approx \boxed{\langle 2.04 , 1.03 \rangle}$