§13.7 SURFACE INTEGRALS

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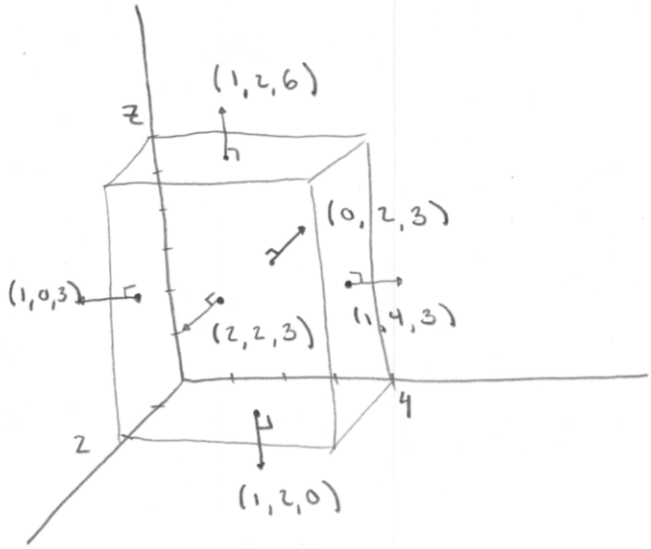
1.



$$\iint_S f\,dS \approx \sum f(P_{ij}^*)\,\Delta S_{ij}$$

AREAS OF CORRESPONDING FACES OF BOX

$$= f(1,2,0)8 + f(2,2,3)24 + f(1,4,3)12$$

$$+ f(0,2,3)24 + f(1,0,3)12 + f(1,2,6)8$$

$$= \boxed{8\left(e^{-.3} + e^{-.9}\right) + 12\left(e^{-.8} + e^{-.4}\right) + 24\left(e^{-.7} + e^{-.5}\right)}$$

4. $$\iint_S f(x,y,z)\,dS = \iint_S g\left(\sqrt{x^2+y^2+z^2}\right)dS = \iint_S g\left(\sqrt{4}\right)dS$$

$= -5\iint_S dS = -5($ SURFACE AREA OF SPHERE OF RADIUS $2)$

$$= -5\,(16\pi) = \boxed{-80\pi}$$

5. $\iint_S (x+y+z)\,dS$

$\vec{r}(u,v) = \langle u+v,\ u-v,\ 1+2u+v \rangle$

$\vec{r}_u = \langle 1, 1, 2 \rangle$, $\vec{r}_v = \langle 1, -1, 1 \rangle$

$|\vec{r}_u \times \vec{r}_v| = \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{matrix} \right\| = |\langle 3, 1, -2 \rangle| = \sqrt{14}$

$= \iint_D f(\vec{r}(u,v))\,|\vec{r}_u \times \vec{r}_v|\,dA = \int_0^1 \int_0^2 (u+v+u-v+1+2u+v)\sqrt{14}\,du\,dv$

$= \sqrt{14} \int_0^1 \int_0^2 4u + v + 1\,du\,dv = \sqrt{14} \int_0^1 \left[2u^2 + uv + u \right]_{u=0}^{u=2} dv$

$= \sqrt{14} \int_0^1 10 + 2v\,dv = \sqrt{14}(10v + v^2)\Big|_0^1 = \boxed{11\sqrt{14}}$

7. $\iint_S y\,dS$, $S: \vec{r}(u,v) = \langle u\cos v,\ u\sin v,\ v \rangle$, $0 \le u \le 1$, $0 \le v \le \pi$

$\vec{r}_u = \langle \cos v, \sin v, 0 \rangle$, $\vec{r}_v = \langle -u\sin v, u\cos v, 1 \rangle$

$|\vec{r}_u \times \vec{r}_v| = \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 1 \end{matrix} \right\| = |\langle \sin v, -\cos v, u \rangle| = \sqrt{1+u^2}$

$\iint_S y\,dS = \int_0^\pi \int_0^1 u \sin v \sqrt{1+u^2}\,du\,dv$

Let $s = 1 + u^2$

$ds = 2u\,du$

$$= \frac{1}{2}\int_0^{\pi}\int_1^{2} \sin v \sqrt{s}\, ds\, dv = \frac{1}{3} s^{3/2}\Big|_1^{2} \int_0^{\pi} \sin v\, dv$$

$$= \frac{1}{3}\left(2^{3/2} - 1\right)\left(-\cos v\right)\Big|_0^{\pi} = \boxed{\frac{2}{3}\left(2^{3/2} - 1\right)}$$

9. $\iint_S x^2 y z \, dS \quad S: z = g(x,y) = 1 + 2x + 3y \;,\; 0 \le x \le 3 \;,\; 0 \le y \le 2$

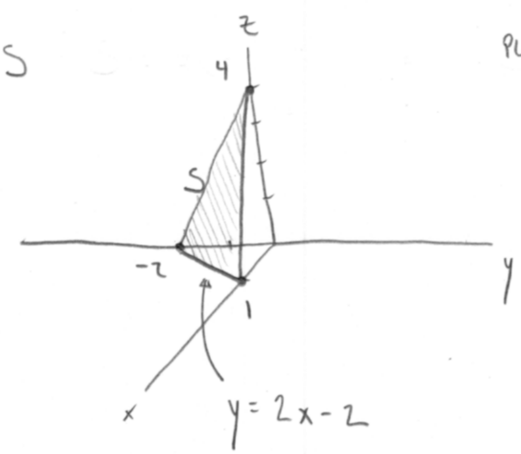
$$dS = \sqrt{g_x^2 + g_y^2 + 1}\; dA = \sqrt{14}\, dA$$

$$= \sqrt{14} \iint_D x^2 y (1 + 2x + 3y)\, dA = \sqrt{14} \int_0^2\int_0^3 x^2 y + 2x^3 y + 3x^2 y^2 \, dx\, dy$$

$$= \sqrt{14}\int_0^2 \frac{1}{3}x^3 y + \frac{1}{2}x^4 y + x^3 y^2 \Big|_{x=0}^{x=3} dy = \sqrt{14}\int_0^2 9y + \frac{81}{2} y + 27y^2 \, dy$$

$$= \sqrt{14}\left(\frac{99}{4}y^2 + 9y^3\right)\Big|_0^2 = \sqrt{14}\,(99 + 72) = \boxed{171\sqrt{14}}$$

11. $\iint_S x \, dS$



PLANE CONTAINS POINT $(1,0,0)$ AND VECTORS $\underbrace{\langle 1, -2, 0\rangle}_{\vec{a}}$ AND $\underbrace{\langle 1, 0, 4\rangle}_{\vec{b}}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 0 \\ 1 & 0 & 4 \end{vmatrix} = \langle -8, -4, 2\rangle$$

$$\langle x-1, y, z\rangle \cdot \langle -8, -4, 2\rangle = 0$$

$$-8x + 8 - 4y + 2z = 0$$

$$z = 4x + 2y - 4 = g(x,y)$$

$$dS = \sqrt{g_x^2 + g_y^2 + 1} = \sqrt{16 + 4 + 1} = \sqrt{21}$$

$$\iint_S x \, dS = \iint_D \sqrt{21}\, x \, dA = \sqrt{21} \int_0^1 \int_{2x-2}^{0} x \, dy \, dx$$

$$= \sqrt{21} \int_0^1 2x - 2x^2 \, dx = \sqrt{21}\left(x^2 - \frac{2}{3}x^3\right)\Big|_0^1 = \sqrt{21}\left(1 - \frac{2}{3}\right) = \boxed{\frac{\sqrt{21}}{3}}$$

13. S: USE CYLINDRICAL COORD: $z = r$, $1 \le r \le 3$, $0 \le \theta \le 2\pi$

$$\iint_S x^2 z^2 \, dS = \int_0^{2\pi}\int_1^3 r^2\cos^2\theta \cdot r^2 \cdot r \, dr \, d\theta = \frac{1}{6}r^6\Big|_1^3 \int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= \frac{1}{6}(729 - 1)\,\frac{1}{2}\int_0^{2\pi} 1 + \cos(2\theta)\, d\theta = \frac{728}{12}\left(\theta + \frac{1}{2}\sin(2\theta)\right)\Big|_0^{2\pi} =$$

$$= \frac{728}{12} \cdot 2\pi = \frac{728\pi}{6} = \boxed{\frac{364\pi}{3}}$$

15. $S: y = g(x,z) = x^2 + z^2 \quad , \quad x^2 + z^2 \le 4$

$$dS = \sqrt{g_x^2 + g_y^2 + 1}\, dA = \sqrt{4x^2 + 4y^2 + 1}\, dA$$

$$\iint_S y\, dS = \iint_D y\sqrt{4x^2+4y^2+1}\, dA = \int_0^{2\pi}\int_0^2 r^2\sqrt{4r^2+1}\cdot r\,dr\,d\theta$$

let $u = 4r^2 + 1$ $\quad \sim \frac{1}{8}\int_0^{2\pi}\int_1^{17} \frac{1}{4}(u-1)\sqrt{u}\, dr\, d\theta$

$du = 8r\,dr$

$$= \frac{2\pi}{32}\int_1^{17} u^{3/2} - u^{1/2}\, du = \frac{\pi}{16}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)\Big|_1^{17}$$

$$= \frac{\pi}{16}\left(\frac{2}{5}\left(17^{5/2} - 1\right) - \frac{2}{3}\left(17^{3/2} - 1\right)\right)$$

$$= \frac{\pi}{8}\left[\sqrt{17}\left(\frac{17^2}{5} - \frac{17}{3}\right) + \frac{1}{3} - \frac{1}{5}\right] = \frac{\pi}{8}\left[\frac{(867-85)\sqrt{17}}{15} + \frac{2}{15}\right]$$

$$= \frac{\pi}{8}\left(\frac{782\sqrt{17} + 2}{15}\right) = \frac{\pi\left(391\sqrt{17} + 1\right)}{60}$$

17. S: SPHERICAL COORD: $\vec{r}(\phi,\theta) = \langle 2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi \rangle$

$0 \le \theta \le 2\pi \quad , \quad 0 \le \phi \le \frac{\pi}{2}$.

$$dS = \rho^2 \sin\phi\, d\phi\, d\theta = 4\sin\phi\, d\phi\, d\theta$$

$$\iint_S x^2 z + y^2 z\, dS = \int_0^{2\pi}\int_0^{\pi/2}\left((2\sin\phi\cos\theta)^2\, 2\cos\phi + (2\sin\phi\sin\theta)^2\, 2\cos\phi\right) 4\sin\phi\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \left[8 \sin^2\phi \cos\phi \left(\cos^2\theta + \sin^2\theta\right)\right] 4 \sin\phi \, d\phi \, d\theta$$

$$= 64\pi \int_0^{\pi/2} \sin^3\phi \cos\phi \, d\phi = 16\pi \sin^4\phi \Big|_0^{\pi/2} = 16\pi$$

19. CYLINDRICAL COORD: $\vec{r}(\theta, x) = \langle x, \cos\theta, \sin\theta \rangle$, $0 \le x \le 3$, $0 \le \theta \le \frac{\pi}{2}$

$\vec{r}_\theta = \langle 0, -\sin\theta, \cos\theta \rangle$, $\vec{r}_x = \langle 1, 0, 0 \rangle$

$$|\vec{r}_\theta \times \vec{r}_x| = \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -\sin\theta & \cos\theta \\ 1 & 0 & 0 \end{matrix} \right\| = |\langle 0, \cos\theta, \sin\theta \rangle|$$

$$= \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

$$\iint_S (z + x^2 y)\, dS = \int_0^{\pi/2}\int_0^3 \sin\theta + x^2 \cos\theta \, dx\, d\theta$$

$$= \int_0^{\pi/2} 3\sin\theta + 9\cos\theta \, d\theta = -3\cos\theta + 9\sin\theta \Big|_0^{\pi/2} = 9 + 3 = \boxed{12}$$

21. $\vec{r}(u,v) = \langle u+v, u-v, 1+2u+v \rangle$, $0 \le u \le 2$, $0 \le v \le 1$

$\vec{r}_u \times \vec{r}_v = \langle 3, 1, -2 \rangle$

NOTE THAT THIS POINTS DOWN SO LET'S TAKE

$-\vec{r}_u \times \vec{r}_v = \langle -3, -1, 2 \rangle$ AS UPWARD NORMAL VECTOR.

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS = \iint_D \vec{F}(\vec{r}) \cdot (-\vec{r}_u \times \vec{r}_v)\, dA$$

$$= \int_0^1 \int_0^2 \langle (1+2u+v)e^{(u+v)(u-v)}, -3(1+2u+v)e^{(u+v)(u-v)}, (u+v)(u-v) \rangle \cdot \langle -3, -1, 2 \rangle \, du\, dv$$

$$= 2 \int_0^1 \int_0^2 u^2 - v^2 \, du\, dv = 2 \int_0^1 \frac{1}{3}u^3 - uv^2 \Big|_{u=0}^{u=2} dv = 2 \int_0^1 \frac{8}{3} - 2v^2 \, dv$$

$$= 2\left(\frac{8}{3}v - \frac{2}{3}v^3\right)\Big|_0^1 = \boxed{4}$$

23. $\vec{r}(x,y) = \langle x, y, g(x,y) \rangle, \quad g(x,y) = 4 - x^2 - y^2$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & g_x \\ 0 & 1 & g_y \end{vmatrix} = \langle -g_x, -g_y, 1 \rangle$$

$$\iint_S \vec{F} \cdot dS = \iint_D \vec{F}(\vec{r}) \cdot (\vec{r}_x \times r_y)\, dA = \iint \langle xy, y(4-x^2-y^2), x(4-x^2-y^2) \rangle \cdot \langle 2x, 2y, 1 \rangle \, dA$$

$$= \int_0^1 \int_0^1 2x^2y + 8y^2 - 2x^2y^2 - 2y^4 + 4x - x^3 - xy^2 \, dx\, dy$$

$$= \int_0^1 \frac{2}{3}y + 8y^2 - \frac{2}{3}y^2 - 2y^4 + 2 - \frac{1}{4} - \frac{1}{2}y^2 \, dy$$

$$= \int_0^1 -2y^4 + \frac{41}{6}y^2 + \frac{2}{3}y + \frac{7}{4} \, dy = -\frac{2}{5} + \frac{41}{18} + \frac{1}{3} + \frac{7}{4}$$

$$= \frac{-72 + 410 + 60 + 315}{180} = \boxed{\frac{713}{180}}$$

25. PARAMETRIZE S: $\vec{r}(\phi, \theta) = \langle 2\sin\phi\cos\theta,\ 2\sin\phi\sin\theta,\ 2\cos\phi \rangle$, $\quad 0 \le \phi \le \frac{\pi}{2}$

$\vec{r}_\phi = \langle 2\cos\phi\cos\theta,\ 2\cos\phi\sin\theta,\ -2\sin\phi \rangle \qquad 0 \le \theta \le \frac{\pi}{2}$

$\vec{r}_\theta = \langle -2\sin\phi\sin\theta,\ 2\sin\phi\cos\theta,\ 0 \rangle$

$$\vec{r}_\phi \times \vec{r}_\theta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2\cos\phi\cos\theta & 2\cos\phi\sin\theta & -2\sin\phi \\ -2\sin\phi\sin\theta & 2\sin\phi\cos\theta & 0 \end{vmatrix}$$

$$= \langle 4\sin^2\phi\cos\theta,\ 4\sin^2\phi\sin\theta,\ 4\sin\phi\cos\phi\cos^2\theta + 4\sin\phi\cos\phi\sin^2\theta \rangle$$

$$= \langle 4\sin^2\phi\cos\theta,\ 4\sin^2\phi\sin\theta,\ 4\sin\phi\cos\phi \rangle$$

NOTE THAT THESE VECTORS POINT OUT OF THE SPHERE SO WE MULTIPLY THIS BY SCALAR -1 TO GET INWARD POINTING NORMAL VECTOR.

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F}(\vec{r}) \cdot (-\vec{r}_\phi \times r_\theta)\, dA$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \langle \overset{x}{2\sin\phi\cos\theta},\ \overset{-z}{-2\cos\phi},\ \overset{y}{2\sin\phi\sin\theta} \rangle \cdot \langle -4\sin^2\phi\cos\theta,\ -4\sin^2\phi\sin\theta,\ -4\sin\phi\cos\phi \rangle\, d\phi\, d\theta$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} -8\sin^3\phi\cos^2\theta + 8\sin^2\phi\cos\phi\sin\theta - 8\sin^2\phi\cos\phi\cos\theta \ d\phi\, d\theta$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} -8(1-\cos^2\phi)\sin\phi\cos^2\theta + 8\sin^2\phi\cos\phi(\sin\theta - \cos\theta)\, d\phi\, d\theta$$

$$= \int_0^{\pi/2} \left(8\left(\cos\phi - \frac{1}{3}\cos^3\phi\right)\Big|_{\phi=0}^{\phi=\frac{\pi}{2}} \cos^2\theta + \frac{8}{3}\sin^3\phi\,\Big|_{\phi=0}^{\phi=\frac{\pi}{2}} (\sin\theta - \cos\theta)\right) d\theta$$

$$= \int_0^{\pi/2} 8\left(0 - 0 - \left(1 - \tfrac{1}{3}\right)\right)\cos^2\theta + \tfrac{8}{3}(\sin\theta - \cos\theta)\,d\theta$$

$$= \int_0^{\pi/2} -\frac{8}{3}(1 + \cos 2\theta) + \frac{8}{3}(\sin\theta - \cos\theta)\,d\theta$$

$$= -\frac{8}{3}\left(\theta + \frac{1}{2}\sin 2\theta\right) + \frac{8}{3}(-\cos\theta - \sin\theta)\Big|_0^{\pi/2}$$

$$= -\frac{8}{3}\left(\frac{\pi}{2}\right) + \frac{8}{3}(1 - 1) = \boxed{-\frac{4\pi}{3}}$$

Alt: $\vec{n} = \dfrac{-1}{\sqrt{x^2+y^2+z^2}}\langle x, y, z\rangle = -\frac{1}{2}\langle x, y, z\rangle$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot\vec{n}\,dS = \iint_S \langle x, -y, z\rangle \cdot -\tfrac{1}{2}\langle x, y, z\rangle\,dS$$

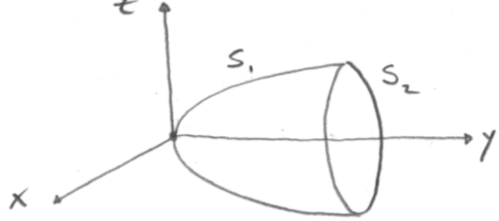
$= -\frac{1}{2}\iint_S x^2\,dS$ NOW USE SAME $\vec{r}$ TO PARAMETRIZE S.

$$= -\frac{1}{2}\iint_D 4\sin^2\phi\cos^2\theta\,dA = -8\int_0^{\pi/2}\int_0^{\pi/2} \sin^3\phi\cos^2\theta\,d\phi\,d\theta$$

$$= -4\int_0^{\pi/2}(1-\cos^2\phi)\sin\phi\,d\theta \int_0^{\pi/2} 1 + \cos 2\theta\,d\theta$$

$$= -4\left[-\cos\phi + \frac{1}{3}\cos^3\phi\right]_0^{\pi/2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$$

$$= -4\left[0 - \left(-1 + \tfrac{1}{3}\right)\right]\left[\frac{\pi}{2}\right] = \boxed{-\frac{4\pi}{3}}$$



27. $\vec{F}(x,y,z) = \langle 0, y, -z \rangle$

$S = S_1 \cup S_2$

$S_1 : \vec{r}(x,z) = \langle x, x^2+z^2, z \rangle \; , \quad x^2+z^2 \le 1$

$S_2 : \vec{r}(x,z) = \langle x, 1, z \rangle \; , \quad x^2+z^2 \le 1$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_{S_1} \vec{F}\cdot d\vec{S} + \iint_{S_2} \vec{F}\cdot d\vec{S}$$

$$\iint_{S_1} \vec{F}\cdot d\vec{S} = \iint_D \vec{F}(\vec{r})\cdot(\vec{r}_x \times \vec{r}_z)\, dS$$

$$= \iint_D \langle 0, x^2+z^2, -z \rangle \cdot \langle 2x, -1, 2z \rangle \, dA$$

$$= \iint_D -x^2 - -3z^2 \, dA \quad \text{POLAR COORD.}$$

$$= \int_0^{2\pi}\int_0^1 \underbrace{\left(-(r\cos\theta)^2 - 3(r\sin\theta)^2\right)}_{-r^2\cos^2\theta - 3r^2\sin^2\theta \,=\, -r^2\left(\underbrace{\cos^2\theta+\sin^2\theta}_{1} + 2\sin^2\theta\right)} r\,dr\,d\theta$$

$$= -\int_0^1 r^3\,dr \int_0^{2\pi} \underbrace{1+2\sin^2\theta}_{1+(1-\cos 2\theta)}\, d\theta = -\frac{1}{4}r^4\Big|_0^1 \left(2\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{2\pi}$$

$$= -\frac{1}{4}(4\pi) = \underline{\underline{-\pi}}$$

$$\iint_{S_2} \vec{F}\cdot d\vec{S} = \iint_{S_2} \vec{F}\cdot\vec{n}\, dS$$ NOTE THAT ON S_2, $\vec{n} = \hat{j} = \langle 0,1,0\rangle$

$$= \iint_{S_2} \langle 0, y, -z\rangle \cdot \langle 0,1,0\rangle\, dS = \iint_S y\, dS = \iint_S 1\, dS$$

$= \underline{\underline{\pi}}$ SINCE AREA OF $S_2 = \pi$.

$$\therefore \iint_S \vec{F}\cdot d\vec{S} = -\pi + \pi = \boxed{0}$$

29. NOTE THAT THE SIX SIDES OF THE CUBE HAVE NORMAL VECTORS $\pm\hat{i}$, $\pm\hat{j}$, $\pm\hat{k}$.

$$\iint_S \vec{F}\cdot d\vec{S} = \sum_{i=1}^{6} \iint_{S_i} \vec{F}\cdot\vec{n}_i\, dS_i$$

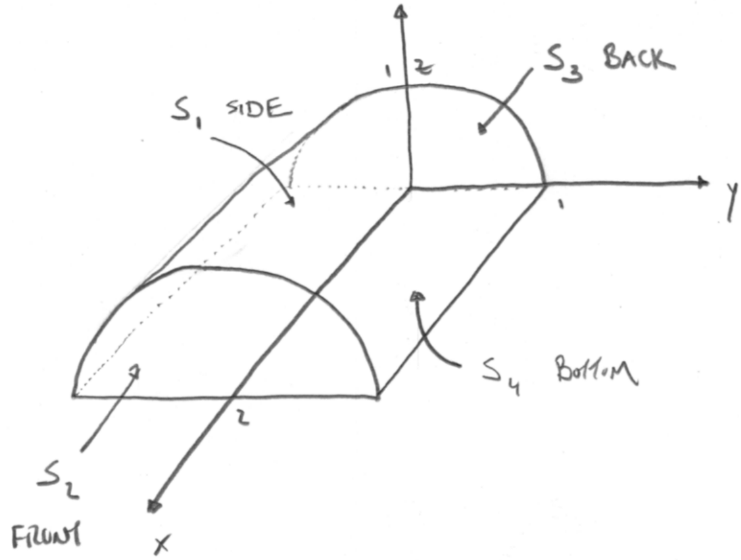
$$= \iint_{S_{x=1}} \langle 1, 2y, 3z\rangle\cdot\langle 1,0,0\rangle\, dS + \iint_{S_{x=-1}} \langle -1, 2y, 3z\rangle\cdot\langle -1,0,0\rangle\, dS$$

$$+ \iint_{S_{y=1}} \langle x, 2, 3z\rangle\cdot\langle 0,1,0\rangle\, dS + \iint_{S_{y=-1}} \langle x, -2, 3\rangle\cdot\langle 0,-1,0\rangle\, dS$$

$$+ \iint_{S_{z=1}} \langle x, 2y, 3\rangle\cdot\langle 0,0,1\rangle\, dS + \iint_{S_{z=-1}} \langle x, 2y, -3\rangle\cdot\langle 0,0,-1\rangle\, dS$$

$$= (1+1+2+2+3+3)\underbrace{\iint_S dS}_{\text{ALL SIDES HAVE AREA } 4} = 12\cdot 4 = \boxed{48}$$

31.



$$\iint_S \vec{F}\cdot d\vec{S} = \sum_{i=1}^{4} \iint_{S_i} \vec{F}\cdot d\vec{S}_i = \sum_{i=1}^{4} \iint_{S_i} \vec{F}\cdot\vec{n}\, dS$$

ON S_1, $\vec{n} = \langle 0, y, z\rangle$

ON S_2, $x = 2$ AND $\vec{n} = \langle 1,0,0\rangle$

ON S_3, $x = 0$ AND $\vec{n} = \langle -1,0,0\rangle$

ON S_4, $z = 0$ AND $\vec{n} = \langle 0,0,-1\rangle$

$$\therefore \iint_S \vec{F}\cdot dS = \iint_{S_1} \langle x^2, y^2, z^2\rangle\cdot\langle 0,y,z\rangle\, dS + \iint_{S_2} \langle 4, y^2, z^2\rangle\cdot\langle 1,0,0\rangle\, dS$$

$$+ \iint_{S_3} \underbrace{\langle 0, y^2, z^2\rangle\cdot\langle -1,0,0\rangle}_{0}\, dS + \iint_{S_4} \underbrace{\langle x^2, y^2, 0\rangle\cdot\langle 0,0,-1\rangle}_{0}\, dS$$

$$= \iint_{S_1} y^3 + z^3\, dS + \underbrace{\iint_{S_2} 4\, dS}_{4 \times \text{AREA OF } S_2 = 4\left(\frac{\pi}{2}\right) = 2\pi}$$

$$\iint_{S_1} y^3 + z^3 \, dS \qquad S_1 : \vec{r}(x,\theta) = \langle x, \cos\theta, \sin\theta \rangle \;,\; 0 \le x \le 2 \;,\; 0 \le \theta \le \pi$$

$$= \iint_D (\cos\theta)^3 + (\sin\theta)^3 \, |\vec{r}_x \times \vec{r}_\theta| \, dA \;,\; |\vec{r}_x \times \vec{r}_\theta| = \left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} \right\|$$

$$= \sqrt{0^2 + \cos^2\theta + \sin^2\theta} = 1$$

$$= \int_0^\pi \int_0^2 \cos^3\theta + \sin^3\theta \, dx \, d\theta = 2 \int_0^\pi (1 - \sin^2\theta)\cos\theta + (1 - \cos^2\theta)\sin\theta \, d\theta$$

$$= 2 \left[\left(\sin\theta - \frac{1}{3}\sin^3\theta \right) \Big|_0^\pi + \left(\frac{1}{3}\cos^3\theta - \cos\theta \right) \Big|_0^\pi \right] = 2 \left[0 + \left(-\frac{1}{3} + 1\right) - \left(\frac{1}{3} - 1\right) \right]$$

$$= 2\left[\frac{4}{3} \right] = \frac{8}{3} \qquad \therefore \iint_S \vec{F} \cdot d\vec{S} = \boxed{\frac{8}{3} + 2\pi}$$