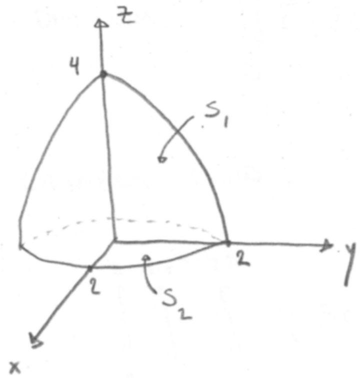
JOHN ADAMSKI
4/21/2017

2.



$$\iint_S \vec{F}\cdot d\vec{S} = \iint_{S_1} \vec{F}\cdot d\vec{S} + \iint_{S_2} \vec{F}\cdot d\vec{S}$$

$$S_1 : \vec{r}(x,y) = \langle x, y, 4-x^2-y^2 \rangle,$$

$$x^2+y^2 \le 2$$

$$\vec{r}_x \times \vec{r}_y = \langle 2x, 2y, 1 \rangle$$

$$\iint_{S_1} \vec{F}\cdot d\vec{S} = \iint_D \langle x^2, xy, 4-x^2-y^2 \rangle \cdot \langle 2x, 2y, 1 \rangle \, dA$$

$$= \iint_D \left(2x^3 + 2xy^2 + 4 - x^2 - y^2\right) dA$$

$x = r\cos\theta$

$y = r\sin\theta$

$$= \int_0^{2\pi}\int_0^2 \underbrace{\left(2r^3\cos^3\theta + 2r^3\sin^2\theta\cos\theta + 4 - r^2\right) r \, dr}_{\frac{2}{5}r^5\cos^3\theta + \frac{2}{5}r^5\sin^2\theta\cos\theta + 2r^2 - \frac{1}{4}r^4 \Big|_{r=0}^{r=2}} \, d\theta$$

$$= \int_0^{2\pi} \left(\underbrace{\frac{64}{5}\cos^3\theta}_{0} + \underbrace{\frac{64}{5}\sin^2\theta\cos\theta}_{0} + 8 - 4\right) d\theta = \underline{\underline{8\pi}}$$

$$\iint_{S_2} \vec{F}\cdot d\vec{S} = \iint_{S_2} \vec{F}\cdot\vec{n}\, dS = \iint_{S_2} \langle x^2, xy, z \rangle \cdot \langle 0,0,-1 \rangle \, dS$$

$$= \iint_{S_2} -z \, dS = \iint_{S_2} 0 \, dS = \underline{\underline{0}}$$

$$\therefore \iint_S \vec{F}\cdot dS = \boxed{8\pi}$$

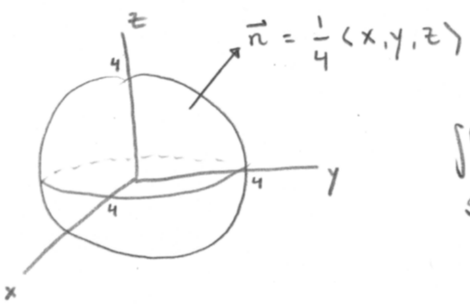
BY DIV. THM. $\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{DIV} \vec{F}\, dV = \iiint_E (3x+1)\, dV$

CYLINDRICAL COORD :

$$= \int_0^2 \int_0^{4-r^2} \int_0^{2\pi} (3r\cos\theta + 1)\, r\, d\theta\, dz\, dr$$

$$= 2\pi \int_0^2 \int_0^{4-r^2} r\, dz\, dr = 2\pi \int_0^2 4r - r^3\, dr = 2\pi \left(2r^2 - \frac{1}{4}r^4\right)\Big|_0^2 = \boxed{8\pi}$$

3.



$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot\vec{n}\, dS$$

$$= \iint_S \langle z,y,x\rangle \cdot \frac{1}{4}\langle x,y,z\rangle\, dS = \frac{1}{4}\iint_S (2xz + y^2)\, dS$$

$x = 4\sin\phi\cos\theta \qquad y = 4\sin\phi\sin\theta \qquad z = 4\cos\phi$

$$= \frac{1}{4}\iint_D \left(32\sin\phi\cos\phi\cos\theta + 16\sin^2\phi\sin^2\theta\right) dA$$

$$= 4\int_0^{\pi}\int_0^{2\pi} (\underbrace{2\sin\phi\cos\phi\cos\theta}_{0} + \sin^2\phi\sin^2\theta)\, 16\sin\phi\, d\theta\, d\phi$$

$$= 64\pi \int_0^{\pi} \sin^3\phi\, d\phi = 64\pi\int_0^{\pi} (1-\cos^2\phi)\sin\phi\, d\phi$$

$$= 64\pi \left(-\cos\phi + \frac{1}{3}\cos^3\phi \right) \Big|_0^{\pi}$$

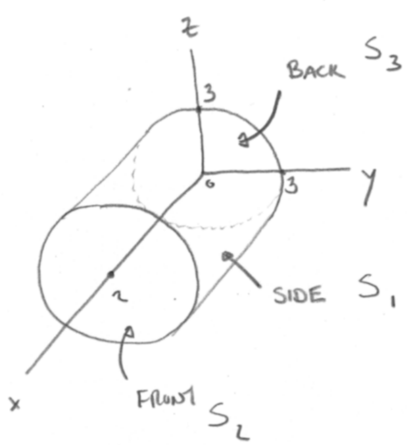
$$= 64\pi \left(2 - \frac{2}{3} \right) = \boxed{\frac{256\pi}{3}}$$

BY DIV THM $\displaystyle\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{DIV}\vec{F}\, dV$

$$= \iiint_E 1\, dV = \text{VOLUME OF BALL OF RADIUS } 4 = \frac{4}{3}\pi(4)^3 = \boxed{\frac{256\pi}{3}}$$

✓

4.



$$\iint_S \vec{F}\cdot d\vec{S} = \iint_{S_1} \vec{F}\cdot d\vec{S} + \iint_{S_2} \vec{F}\cdot d\vec{S} + \iint_{S_3} \vec{F}\cdot d\vec{S}$$

$$\iint_{S_1} \vec{F}\cdot d\vec{S} = \iint_{S_1} \vec{F}\cdot\vec{n}\, dS = \iint_{S_1} \langle x^2, -y, z\rangle \cdot \frac{1}{3}\langle 0, y, z\rangle\, dS$$

$$= \frac{1}{3}\iint \left(-y^2 + z^2\right) dS$$

CYLINDRICAL COORD:

$x = x \qquad y = 3\cos\theta \qquad z = 3\sin\theta$

$$= 3\iint_D \sin^2\theta - \cos^2\theta\, dA = -3\int_0^2\int_0^{2\pi} \cos 2\theta\, d\theta\, dx = \underline{\underline{0}}$$

$$\iint_{S_2} \vec{F}\cdot d\vec{S} = \iint_{S_2} \vec{F}\cdot\vec{n}\, dS = \iint_{S_2} \langle x^2, -y, z\rangle \cdot \langle 1, 0, 0\rangle \, dS$$

$$= \iint_{S_2} x^2 \, dS = 4 \underbrace{\iint_{S_2} dS}_{\text{AREA OF } S_2} = 4(9\pi) = \underline{\underline{36\pi}}$$

$$\iint_{S_3} \vec{F}\cdot d\vec{S} = \iint_{S_2} \langle x^2, -y, z\rangle \cdot \langle -1, 0, 0\rangle \, dS = \iint_{S_2} -x^2 \, dS = \iint_{S_2} 0 \, dS = \underline{\underline{0}}$$

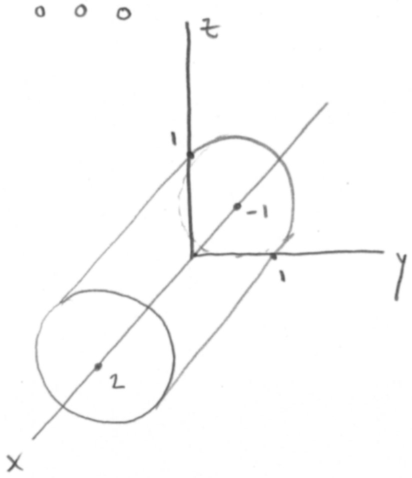
$$\therefore \iint_{S} \vec{F}\cdot d\vec{S} = 0 + 36\pi + 0 = \boxed{36\pi}$$

BY DIV THM, $\iint_{S} \vec{F}\cdot d\vec{S} = \iiint_{E} \text{DIV}\, \vec{F} \, dV = \iiint_{E} 2x \, dV$

$x = x \qquad y = r\cos\theta \qquad z = r\sin\theta$

$$2 \int_0^{2\pi}\int_0^{3}\int_0^{2} x r \, dx\, dr\, d\theta = 8\pi \int_0^3 r\, dr = 4\pi r^2 \Big|_0^3 = \boxed{36\pi}$$

7.



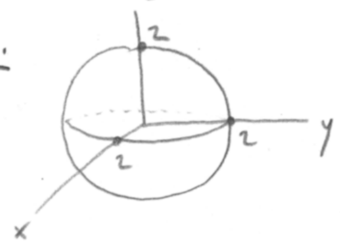
$$\iint_{S} \vec{F}\cdot d\vec{S} = \iiint_{E} \text{DIV}\, \vec{F} \, dV$$

$$= \iiint_{E} \left(3y^2 + 3z^2\right) dV$$

$x = x \qquad y = r\cos\theta \qquad z = r\sin\theta$

$$= 3\int_0^{2\pi}\int_0^{1}\int_{-1}^{2} r^3\,dx\,dr\,d\theta = 18\pi\int_0^1 r^3\,dr = \frac{9}{2}\pi r^4\Big|_0^1 = \boxed{\frac{9}{2}\pi}$$

8.



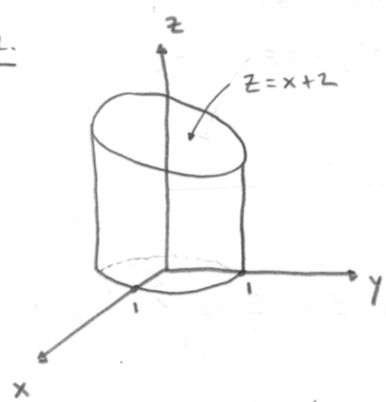
$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \text{DIV}\,\vec{F}\,dV = \iiint_E \left(3x^2+3y^2+3z^2\right)dV$$

$$= 3\int_0^{2\pi}\int_0^{\pi}\int_0^{2} \rho^2\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{192\pi}{5}\int_0^{\pi}\sin\phi\,d\phi = \boxed{\frac{384\pi}{5}}$$

9.

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \text{DIV}\,\vec{F}\,dV = \iiint 0\,dV = \boxed{0}$$

12.



$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \text{DIV}\,\vec{F}\,dV$$

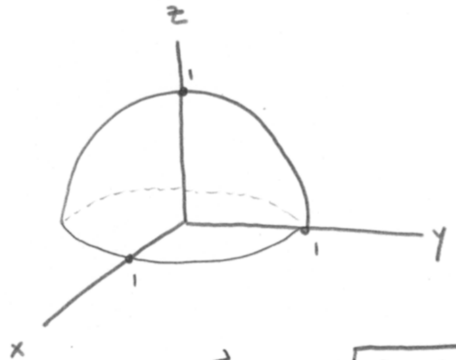
$$= \iiint_E \underbrace{4x^3 + 4xy^2}_{4x(x^2+y^2)}\,dV$$

$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$

$$= 4\int_0^{2\pi}\int_0^{1}\int_0^{r\cos\theta+2} r^4\cos\theta\,dz\,dr\,d\theta = 4\int_0^{2\pi}\int_0^{1} \underbrace{r^4\cos\theta\left(r\cos\theta+2\right)}_{r^5\cos^2\theta + 2r^4\cos\theta}\,dr\,d\theta$$

$$= 4\int_0^{2\pi} \frac{1}{6}\cos^2\theta + \cancel{\frac{2}{5}\cos\theta}\ d\theta = \boxed{\frac{2\pi}{3}}$$

13.



$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{DIV}\vec{F}\,dV$$

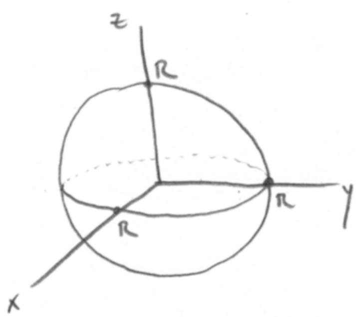
$$\vec{F} = \left\langle x\sqrt{x^2+y^2+z^2},\ y\sqrt{x^2+y^2+z^2},\ z\sqrt{x^2+y^2+z^2}\right\rangle$$

$$\operatorname{DIV}\vec{F} = 3\sqrt{x^2+y^2+z^2} + \frac{x^2+y^2+z^2}{\sqrt{x^2+y^2+z^2}} = 4\sqrt{x^2+y^2+z^2}$$

$$= 4\iiint_E \sqrt{x^2+y^2+z^2}\,dV = 4\int_0^{2\pi}\int_0^{\pi/2}\int_0^1 \rho\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= 2\pi\int_0^{\pi/2}\sin\phi\,d\phi = \boxed{2\pi}$$

14.



$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{DIV}\vec{F}\,dV$$

$$\vec{F} = \left\langle x(x^2+y^2+z^2),\ y(x^2+y^2+z^2),\ z(x^2+y^2+z^2)\right\rangle$$

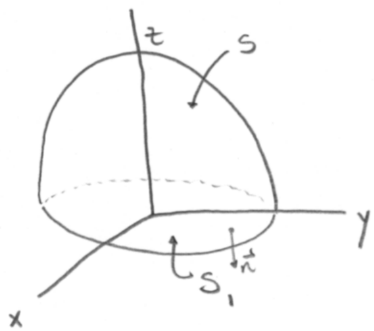
$$\operatorname{DIV}\vec{F} = 3(x^2+y^2+z^2) + 2x^2 + 2y^2 + 2z^2 = 5(x^2+y^2+z^2)$$

$$\iiint_E \operatorname{DIV}\vec{F}\,dV = 5\iiint_E x^2+y^2+z^2\,dV$$

$$= 5 \int_0^{2\pi} \int_0^{\pi} \int_0^{1} \rho^2 \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= 2\pi \int_0^{\pi} \sin\phi \, d\phi = \boxed{4\pi}$$

17.



$S_2 = S \cup S_1$

$$\iint_{S_2} \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F} \, dV$$

$$\iint_S \vec{F} \cdot d\vec{S} + \iint_{S_1} \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F} \, dV$$

$$\therefore \iint_S \vec{F} \cdot d\vec{S} = \underbrace{\iiint_E \operatorname{div} \vec{F} \, dV}_{(i)} - \underbrace{\iint_{S_1} \vec{F} \cdot d\vec{S}}_{(ii)}$$

$$(i) = \iiint_E (x^2 + y^2 + z^2) \, dV = \int_0^{2\pi} \int_0^{\pi/2} \int_0^{1} \rho^2 \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \frac{2\pi}{5} \int_0^{\pi/2} \sin\phi \, d\phi = \underline{\underline{\frac{2\pi}{5}}}$$

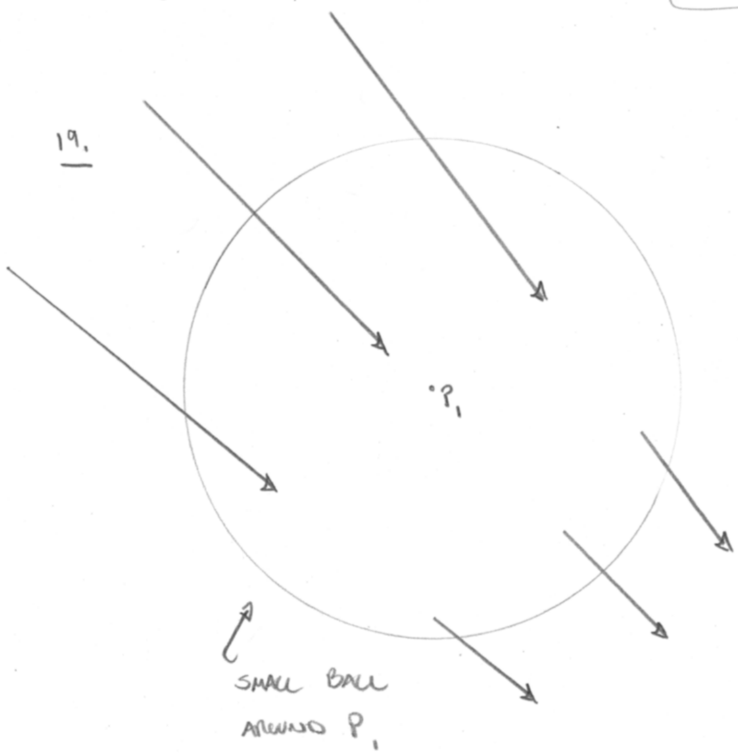
$$(ii) = \iint_{S_1} \vec{F} \cdot \vec{n} \, dS = \iint_{S_1} \langle z^2 x, \tfrac{1}{3} y^3 + \tan z, x^2 z + y^2 \rangle \cdot \langle 0, 0, -1 \rangle \, dS$$

$$= -\iint_{S_1} (x^2 \underset{z=0 \text{ on } S_1}{z} + y^2) \, dS = -\int_0^{2\pi} \int_0^{1} r^3 \sin^2\theta \, dr \, d\theta = -\frac{1}{4} \int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= \underline{\underline{-\frac{\pi}{4}}}$$

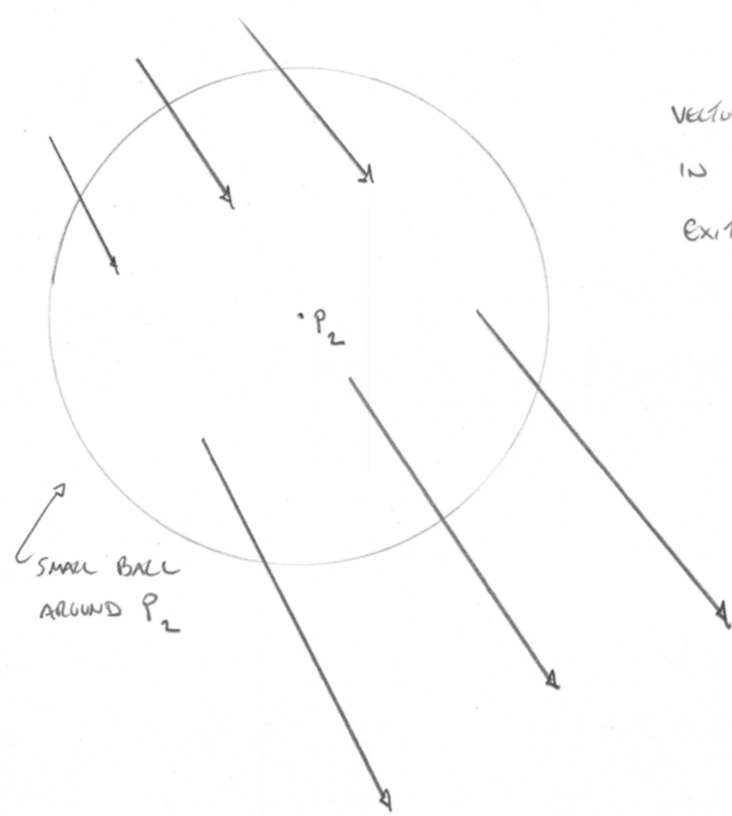
$$\therefore \iint_S \vec{F} \cdot d\vec{S} = \frac{2\pi}{5} - \left(-\frac{\pi}{4}\right) = \boxed{\frac{13\pi}{20}}$$

19.



VECTORS COMING IN ARE LARGER IN MAGNITUDE THAN VECTORS EXITING

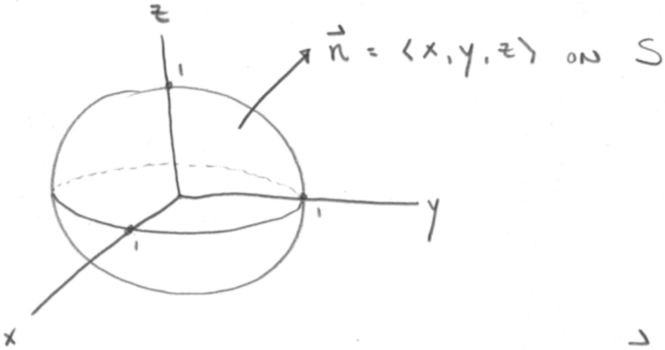
$\Rightarrow$ NEGATIVE DIVERGENCE



VECTORS COMING IN ARE SMALLER IN MAGNITUDE THAN VECTORS EXITING

$\Rightarrow$ POSITIVE DIVERGENCE

24.



$$\iint_S (2x + 2y + z^2)\, dS = \iint_S \overbrace{\langle 2, 2, z \rangle}^{\vec{F}} \cdot \overbrace{\langle x, y, z \rangle}^{\vec{n}}\, dS$$

$$= \iint_S \vec{F} \cdot d\vec{S} = \iiint_E \text{DIV}\, \vec{F}\, dV = \iiint_E 1\, dV = \boxed{\frac{4}{3}\pi}$$

25. $$\iint_S \vec{a} \cdot \vec{n}\, dS = \iiint_E \text{DIV}\, \vec{a}\, dV = \iiint_E 0\, dV = 0 \quad \checkmark$$

27. $$\iint_S \text{CURL}\, \vec{F} \cdot d\vec{S} = \iiint_E \text{DIV}\left(\text{CURL}\, \vec{F}\right) dV$$

$$= \iiint_E 0\, dV = 0 \quad \checkmark$$

(DIV (CURL $\vec{F}$) = 0 FOR ALL $\vec{F}$ WITH CONTINUOUS 2^{nd} PARTIAL DERIVATIVES)