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MATH 392

CH. 1 : MATRICES & MATRIX ALGEBRA

1

$$2\begin{bmatrix}1 & 2 & 5\\3 & 4 & 0\end{bmatrix} - 3\begin{bmatrix}0 & 2 & 1\\1 & 4 & -3\end{bmatrix} = \begin{bmatrix}2 & 4 & 10\\6 & 8 & 0\end{bmatrix} + \begin{bmatrix}0 & -6 & -3\\-3 & -12 & 9\end{bmatrix}$$

$$= \boxed{\begin{bmatrix}2 & -2 & 7\\3 & -4 & 9\end{bmatrix}}$$

2

$$\underbrace{\begin{bmatrix}1 & 2 & 1\\-1 & 6 & 4\\3 & 1 & 2\end{bmatrix}}_{(3\times 3)}\underbrace{\begin{bmatrix}1 & 1\\0 & -1\\2 & 3\end{bmatrix}}_{(3\times 2)} = \underbrace{\boxed{\begin{bmatrix}3 & 2\\7 & 5\\7 & 8\end{bmatrix}}}_{(3\times 2)}$$

CONFORMABLE ✓

3

$$\underbrace{\begin{bmatrix}3 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}}_{(3\times 3)}\underbrace{\begin{bmatrix}2 & 1 & 1 & 5\\1 & 0 & 1 & 3\\2 & 4 & 1 & 5\end{bmatrix}}_{(3\times 4)} = 3\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 1 & 5\\1 & 0 & 1 & 3\\2 & 4 & 1 & 5\end{bmatrix}$$

✓

$$= 3\begin{bmatrix}2 & 1 & 1 & 5\\1 & 0 & 1 & 3\\2 & 4 & 1 & 5\end{bmatrix} = \underbrace{\boxed{\begin{bmatrix}6 & 3 & 3 & 15\\3 & 0 & 3 & 9\\6 & 12 & 3 & 15\end{bmatrix}}}_{(3\times 4)}$$

4

$$\begin{bmatrix}\alpha & 0 & 0\\0 & \alpha & 0\\0 & 0 & \alpha\end{bmatrix}\begin{bmatrix}1 & 6 & 5\\3 & 0 & 1\\2 & 1 & 1\end{bmatrix} = \alpha I\begin{bmatrix}1 & 6 & 5\\3 & 0 & 1\\2 & 1 & 1\end{bmatrix}$$

$$= \alpha\begin{bmatrix}1 & 6 & 5\\3 & 0 & 1\\2 & 1 & 1\end{bmatrix} = \boxed{\begin{bmatrix}\alpha & 6\alpha & 5\alpha\\3\alpha & 0 & \alpha\\2\alpha & \alpha & \alpha\end{bmatrix}}$$

5

$$\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}\begin{bmatrix} 1 & 5 & 3 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} \alpha & 5\alpha & 3\alpha & 2\alpha \\ 0 & \beta & \beta & \beta \end{bmatrix}}$$

(2×2) ✓ (2×4) (2×4)

6

$$\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ -4 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & 0 \\ -4 & 0 \end{bmatrix}}.$$

$$\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}\right) = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 6 & 0 \\ -4 & 0 \end{bmatrix}}.$$ ✓

THIS VERIFIES THE DISTRIBUTIVE PROPERTY.

7

$$AI_3 = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 4 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 4 & 6 \end{bmatrix} = \boxed{A}$$

$$I_2A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 5 \\ -2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 4 & 6 \end{bmatrix} = \boxed{A}$$ ✓

9

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -11 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\Rightarrow \begin{bmatrix} -11a + 4b & -3a + b \\ -11c + 4d & -3c + d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

SO $-3a + b = 0 \Rightarrow b = 3a$

THEN $-11a + 4b = 1 \Rightarrow -11a + 12a = a = 1$, & so $b = 3$.

NOW $-3c + d = 1 \Rightarrow d = 1 + 3c$

THEN $-11c + 4d = 0 \Rightarrow -11c + 4(1+3c) = c + 4 = 0 \Rightarrow c = -4$,

AND SO $d = 1 + 3(-4) = -11$

$$\therefore \boxed{\begin{array}{l} a = 1,\ b = 3,\ c = -4,\ d = -11 \\ \text{i.e. } \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -4 & -11 \end{bmatrix} \end{array}}$$

$\longleftarrow$ COMPARE TO $\begin{bmatrix} -11 & -3 \\ 4 & 1 \end{bmatrix}$

10

$$\begin{aligned} y &= 2w + x \\ z &= w + x \end{aligned} \longrightarrow \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} w \\ x \end{bmatrix}$$

$$\begin{aligned} u &= 4y \\ v &= 3y - 4z \end{aligned} \longrightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}$$

$$\therefore \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} w \\ x \end{bmatrix} = \boxed{\begin{bmatrix} 8 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} w \\ x \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}}$$

11 (a) $a_{32} = 3$ (b) $a_{23} = -3$ (c) 3×4 (d) $a_2 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

(e) $a_3 = \begin{bmatrix} 5 & 3 & -1 & 2 \end{bmatrix}$ (f) $A^T = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 3 \\ 2 & -3 & -1 \\ 3 & 1 & 2 \end{bmatrix}$

12 IF $a = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ & $b = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

THEN $a \cdot b = a^T b = \begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \boxed{-2}$

13 $$\begin{bmatrix} -1 & 3 & 7 \\ 5 & 1 & 4 \end{bmatrix} + \underbrace{\begin{bmatrix} 2x_{11} & 2x_{12} & 2x_{13} \\ 2x_{21} & 2x_{22} & 2x_{23} \end{bmatrix}}_{(2X)} = \begin{bmatrix} 5 & 1 & 11 \\ 9 & 3 & 12 \end{bmatrix}$$

$$\Rightarrow \boxed{X = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 4 \end{bmatrix}}$$

14 (a) $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ -3 & -2 & -1 \\ 3 & 2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 5 & 6 & 7 \\ -2 & 0 & -2 \\ 2 & 0 & -2 \end{bmatrix}}$$

(b) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 1 + 2 = \boxed{3}$

15 (a) $P(A|B) = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}\left[\begin{array}{cc|cc} 1 & 1 & 2 & -1 \\ 1 & -1 & 1 & 3 \end{array}\right]$

$$= \left[\begin{array}{cc|cc} 3 & -1 & 4 & 5 \\ 2 & 4 & 5 & -6 \end{array}\right] = (PA|PB) \checkmark$$

(b) $P(A|B) = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & -1 & 0 & 3 \\ 1 & 1 & 1 & -2 & -3 & 0 \end{array}\right]$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 5 & 5 \\ 2 & 2 & 2 & -4 & -4 & 4 \\ 4 & 4 & 4 & -3 & -1 & 7 \end{array}\right] = (PA|PB) \checkmark$$