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7/4/2012

MATH 392

CH 2 : SOLVING A SYSTEM WITH ROW OPERATIONS

[1] (a) $A = \begin{bmatrix} 2 & 3 & 5 & 7 \end{bmatrix}$, $(A|b) = \left[\begin{array}{cccc|c} 2 & 3 & 5 & 7 & 8 \end{array}\right]$

(b) $A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \\ 5 & 4 \\ 6 & 5 \end{bmatrix}$, $(A|b) = \left[\begin{array}{cc|c} 3 & 2 & 1 \\ 4 & 3 & 2 \\ 5 & 4 & 3 \\ 6 & 5 & 4 \end{array}\right]$

(c) $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$, $(A|b) = \left[\begin{array}{ccc|c} 2 & 2 & -1 & -1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{array}\right]$

(d) $A = \begin{bmatrix} 2 & 1 & -4 & 4 \\ 1 & 2 & 1 & 4 \end{bmatrix}$, $(A|b) = \left[\begin{array}{cccc|c} 2 & 1 & -4 & 4 & 1 \\ 1 & 2 & 1 & 4 & 2 \end{array}\right]$

(e) $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 5 \end{bmatrix}$, $(A|b) = \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 1 & 3 & 0 \\ 2 & 5 & 0 \end{array}\right]$

(f) $A = \begin{bmatrix} 3 & 5 \\ 2 & 7 \end{bmatrix}$, $(A|b) = \left[\begin{array}{cc|c} 3 & 5 & 4 \\ 2 & 7 & 1 \end{array}\right]$

$\boxed{2}$ (a) $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 3 & 2 & 4 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 3 \end{bmatrix} \overset{3}{\sim} \boxed{\begin{bmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 1 & 3 \end{bmatrix}}$

(b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & 3 \\ 3 & 4 & 2 & 2 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 \end{bmatrix}$

$\overset{2}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \overset{3}{\sim} \boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}}$

(c) $\begin{bmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{bmatrix} \overset{1}{\sim} \begin{bmatrix} 1 & 2 & -2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 1 & 2 & -2 \\ 0 & 6 & -3 \\ 0 & -3 & 6 \end{bmatrix}$

$\overset{1}{\sim} \begin{bmatrix} 1 & 2 & -2 \\ 0 & -3 & 6 \\ 0 & 6 & -3 \end{bmatrix} \overset{2}{\sim} \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & 6 & -3 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 9 \end{bmatrix}$

$\overset{2}{\sim} \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \overset{3}{\sim} \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$

(d)

$$\begin{bmatrix} 2 & 2 & -1 & -1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \overset{1}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & -1 & -1 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -3 & -3 \end{bmatrix}$$

$$\overset{3}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \overset{3}{\sim} \boxed{\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

(e)

$$\begin{bmatrix} 2 & 2 & -1 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \overset{1}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & -1 & 1 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -3 & -1 \end{bmatrix}$$

$$\overset{3}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \overset{2}{\sim} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \overset{3}{\sim} \boxed{\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

(f)

$$\begin{bmatrix} 0 & 2 & -1 & 1 \\ 0 & 1 & 1 & 5 \end{bmatrix} \overset{1}{\sim} \begin{bmatrix} 0 & 1 & 1 & 5 \\ 0 & 2 & -1 & 1 \end{bmatrix} \overset{3}{\sim} \begin{bmatrix} 0 & 1 & 1 & 5 \\ 0 & 0 & -3 & -9 \end{bmatrix}$$

$$\overset{2}{\sim} \begin{bmatrix} 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix} \overset{3}{\sim} \boxed{\begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}}$$

$\boxed{3}$ (a) $x = 0$
$y = 1$

(b) $x = 1$
$2x = 0$

(c)
$$\begin{aligned} x + 2y - z &= 1 \\ 3x - y &= 2 \\ y + z &= -1 \end{aligned}$$

(d)
$$\begin{aligned} x - y &= 2 \\ 2x &= -1 \\ x + y &= 0 \end{aligned}$$

$\boxed{4}$

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

ENTRIES BELOW THIS PIVOT ARE NOT ALL ZERO.

$$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

THIS PIVOT IS NOT 1.

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

ZERO ROWS ARE SUPPOSED TO BE THE BOTTOM ROWS.

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 2 & -1 \\ 1 & 0 & 0 & 1 & 2 \end{bmatrix}$$

THE PIVOT IN THE K^{TH} ROW MUST LIE TO THE RIGHT OF THE PIVOTS IN ROWS $1-(K-1)$.

$\boxed{5}$ THE TWO SYSTEMS ARE EQUIVALENT IF THE ROW-ECHELON FORM OF THEIR AUGMENTED MATRICES ARE EQUAL. WE NOW COMPUTE THOSE.

SYSTEM 1:
$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 0 \\ -1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 2 & -1 \end{array}\right]$$

$$\overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & 2 & -2 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 4 & -4 \end{array}\right] \overset{2}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$$\overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

DONE.

SYSTEM 2:
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 2 & 1 & -1 & 0 & -1 \\ 1 & 0 & -1 & -1 & 0 \\ -1 & 1 & 1 & 0 & 2 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & -1 & 4 & -3 \\ 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 1 & -2 & 3 \end{array}\right]$$

$$\overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & -1 & 4 & -3 \\ 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 2 & -6 & 6 \end{array}\right] \overset{2}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & -1 & 4 & -3 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -3 & 3 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & -1 & 4 & -3 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -2 & 2 \end{array}\right]$$

$$\sim_2 \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & -1 & 4 & -3 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim_3 \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim_3 \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

DONE.

THEY ARE THE SAME ✓. ∴ THE SYSTEMS ARE EQUIVALENT.

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$$\begin{bmatrix} 0 & 1 & 1 & 5 \\ 0 & 1 & 2 & 7 \\ 0 & 1 & 2 & 7 \end{bmatrix} \sim_3 \begin{bmatrix} 0 & 1 & 1 & 5 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim_3 \begin{bmatrix} 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim_3 \boxed{\begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

7 3 NON-ZERO ROWS $\Rightarrow$ 3 PIVOTS

AND EACH COLUMN WHICH CONTAINS A PIVOT WILL HAVE ALL OTHER ENTRIES EQUAL TO 0.

THERE IS ONLY ONE POSSIBILITY: $\boxed{\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}$

NOTE: COLUMNS OF 0'S WILL STAY COLUMNS OF ZEROS THROUGHOUT ANY SEQUENCE OF ROW OPERATIONS.