MATH 392

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CH 3

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1 (d)
$$
\begin{aligned}
a + b - c &= 1 \\
+ \quad -a - b + c &= 2 \\
\hline
0 &= 3
\end{aligned}
$$
(INCONSISTENT)

(f)
$$
\left[\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ 1 & 1 & -1 & 0 \\ 1 & -4 & 2 & -1 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 0 & -5 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & -1 \end{array}\right]
$$

ROW 1 $\Rightarrow$ $-5b + 3c = 1$
ROW 3 $\Rightarrow$ $-5b + 3c = -1$
CONTRADICTION (INCONSISTENT)

(g)
$$
\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & -2 & 2 & 1 \\ 3 & 1 & -1 & 1 & 1 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & -2 & 2 & 1 \\ 4 & 0 & 0 & 0 & 1 \end{array}\right]
$$

$$
\overset{2}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & -2 & 2 & 1 \\ 2 & 0 & 0 & 0 & 1/2 \end{array}\right]
$$
INCONSISTENT

$\boxed{2}$

(c) $\begin{bmatrix} 3 & -1 & 0 \\ 2 & 5 & 0 \end{bmatrix} \overset{3}{\sim} \left[\begin{array}{cc|c} 1 & -6 & 0 \\ 2 & 5 & 0 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cc|c} 1 & -6 & 0 \\ 0 & 17 & 0 \end{array}\right]$

$\overset{2,3}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \quad \therefore \quad \boxed{\begin{array}{l} x = y \\ y = 0 \end{array}}$

RANK = 2

(f) $\left[\begin{array}{ccc|c} 2 & -1 & 2 & -1 \\ 1 & -2 & 3 & -4 \\ 3 & 2 & 2 & 3 \end{array}\right] \overset{1}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 2 & -1 & 2 & -1 \\ 3 & 2 & 2 & 3 \end{array}\right]$

$\overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 3 & -4 & 7 \\ 0 & 8 & -7 & 15 \end{array}\right] \overset{2}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 9 & -12 & 21 \\ 0 & 8 & -7 & 15 \end{array}\right]$

$\overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 1 & -5 & 6 \\ 0 & 8 & -7 & 15 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 1 & -5 & 6 \\ 0 & 0 & 33 & -33 \end{array}\right] \quad \boxed{\begin{array}{l} x = 1 \\ y = 1 \\ z = -1 \end{array}}$

$\overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 1 & -5 & 6 \\ 0 & 0 & 1 & -1 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$

RANK = 3

(j)

$$\left[\begin{array}{cccc|c} -2 & 2 & -1 & 0 & -4 \\ 0 & 5 & 1 & -2 & -3 \\ 2 & -1 & 2 & 1 & 5 \\ 0 & 2 & 1 & -1 & 0 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} -2 & 2 & -1 & 0 & -4 \\ 0 & 5 & 1 & -2 & -3 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & -1 & 0 \end{array}\right]$$

$$\overset{1}{\sim} \left[\begin{array}{cccc|c} -2 & 2 & -1 & 0 & -4 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 5 & 1 & -2 & -3 \\ 0 & 2 & 1 & -1 & 0 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} -2 & 2 & -1 & 0 & -4 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -4 & -7 & -8 \\ 0 & 0 & -1 & -3 & -2 \end{array}\right]$$

$$\overset{1,2}{\sim} \left[\begin{array}{cccc|c} -2 & 2 & -1 & 0 & -4 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & -4 & -7 & -8 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} -2 & 2 & -1 & 0 & -4 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 5 & 0 \end{array}\right]$$

$$\overset{2,3}{\sim} \left[\begin{array}{cccc|c} -2 & 2 & -1 & 0 & -4 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} -2 & 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\overset{3,2}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

RANK = 2

$$\boxed{\begin{array}{l} x_1 = 0 \\ x_2 = -1 \\ x_3 = 2 \\ x_4 = 0 \end{array}}$$

3 (d) $\left[\begin{array}{ccc|c} 1 & -3 & +1 & 4 \\ -2 & -19 & +3 & -3 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -25 & 5 & 5 \end{array}\right]$

$\overset{2}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & 1 & -\frac{1}{5} & -\frac{1}{5} \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{5} & \frac{17}{5} \\ 0 & 1 & -\frac{1}{5} & -\frac{1}{5} \end{array}\right]$

$x = \frac{17}{5} - \frac{2}{5} z$

$y = -\frac{1}{5} + \frac{1}{5} z$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 17 \\ -1 \\ 0 \end{bmatrix} + \frac{\alpha}{5}\begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}}$$

(f) $\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -1 \\ 2 & 3 & -3 & 2 & -4 \\ -1 & -2 & 2 & -1 & 3 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 0 & -2 \\ 0 & -1 & 1 & 0 & 2 \end{array}\right]$

$\overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore \quad x_1 = 1 - x_4$

$x_2 = -2 + x_3$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} + \alpha\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \beta\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}}$$

(g)

SAME

$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & -2 & -1 & 1 & -1 & 0 \\ -1 & 5 & 1 & -2 & -1 & 0 \\ -1 & -7 & -2 & 1 & 2 & 0 \\ \cancel{1} & \cancel{7} & \cancel{2} & \cancel{-1} & \cancel{-2} & \cancel{0} \\ -2 & -8 & -2 & 1 & 3 & 0 \end{array}\right] \overset{1,3}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -4 & -3 & -1 & -1 & 0 \\ 0 & 6 & 2 & -1 & -1 & 0 \\ 0 & -6 & -1 & 2 & 2 & 0 \\ 0 & -6 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\overset{2,3}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & -5 & -5 & 0 \\ 0 & 6 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \overset{1,3}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 6 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\overset{3,2}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 6 & 0 & -3 & -3 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \overset{2,3}{\sim} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\therefore \quad x_1 = -\frac{1}{2}x_4 + \frac{1}{2}x_5$$

$$x_2 = \frac{1}{2}x_4 + \frac{1}{2}x_5$$

$$x_3 = -x_4 - x_5$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \alpha \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ -1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -1 \\ 0 \\ 1 \end{bmatrix}}$$

$\boxed{4}$ (a) $$\left[\begin{array}{ccc|c} 3 & 1 & -2 & 0 \\ -2 & -1 & 1 & 1 \\ -1 & -2 & -1 & -4 \end{array}\right] \overset{1,2}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ -2 & -1 & 1 & 1 \\ 3 & 1 & -2 & 0 \end{array}\right]$$

$$\overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 3 & 3 & 9 \\ 0 & -5 & -5 & -12 \end{array}\right] \overset{2,3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

INCONSTENT $\Rightarrow$ $\boxed{\text{NO SOL'NS}}$

(d) $$\left[\begin{array}{ccc|c} 2 & -1 & 3 & -1 \\ 1 & 1 & 1 & 0 \\ -1 & 2 & -3 & 2 \end{array}\right] \overset{1,3}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & 1 & -1 \\ 0 & 3 & -2 & 2 \end{array}\right]$$

$$\overset{2}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 3 & -2 & 2 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & \frac{4}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 0 & -1 & 1 \end{array}\right]$$

$$\overset{2,3}{\sim}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \therefore \boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}$$

(e) $$\left[\begin{array}{ccccc|c} 1 & 1 & 3 & 0 & 1 & 0 \\ 1 & -1 & -1 & -2 & -1 & 0 \\ -1 & 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 1 & -2 & 0 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & 3 & 0 & 1 & 0 \\ 0 & -2 & -4 & -2 & -2 & 0 \\ 0 & 2 & 4 & 2 & 2 & 0 \\ 0 & 1 & -2 & 1 & -3 & 0 \end{array}\right]$$

$$\overset{1,3}{\sim}\left[\begin{array}{ccccc|c} 1 & 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & -2 & 1 & -3 & 0 \\ 0 & 0 & 8 & 0 & 8 & 0 \\ 0 & 0 & -8 & 0 & -8 & 0 \end{array}\right] \overset{2,3}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\overset{3}{\sim}\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \therefore \begin{aligned} x_1 &= x_4 + x_5 \\ x_2 &= -x_4 + x_5 \\ x_3 &= -x_5 \end{aligned}$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}}$$

5

$$a(0) + b(0) + c(-1) + d = 0$$
$$a(1) + b(1) + c(4) + d = 0$$
$$a(-1) + b(1) + c(0) + d = 0$$

$$\rightarrow \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & 4 & 1 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \left[\begin{array}{cccc|c} 0 & 0 & -1 & 1 & 0 \\ 1 & 1 & 4 & 1 & 0 \\ -1 & 1 & 0 & 1 & 0 \end{array}\right] \overset{1,3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 4 & 1 & 0 \\ 0 & 2 & 4 & 2 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{array}\right]$$

$$\overset{2}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 5 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$$\overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right] \quad \therefore \quad \begin{aligned} a &= -2d \\ b &= -3d \\ c &= d \end{aligned}$$

$$\therefore (-2d)x + (-3d)y + (d)z + d = 0$$

$$\therefore \boxed{-2x - 3y + z + 1 = 0}$$

$\boxed{6}$ (a) $\left[\begin{array}{cc|c} 3 & 6 & 3 \\ 6 & 12 & 12 \end{array}\right] \overset{2,3}{\sim} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 6 \end{array}\right]$

INCONSISTENT

$\boxed{\text{NO SOL'N}}$

(b) $\left[\begin{array}{cc|c} 3 & 6 & -3 \\ 6 & 12 & -6 \end{array}\right] \overset{2,3}{\sim} \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 0 & 0 \end{array}\right] \therefore \; x = -1 - 2y$

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} -2 \\ 1 \end{bmatrix}}$$

(d) $\left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ -1 & 1 & 1 & -3 \\ 1 & 5 & -7 & 3 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ 0 & 3 & -3 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right]$

$\overset{3,2}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & -4 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore \quad x = 3 + 2z$

$y = z$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}}$$

(e) $\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 1 & 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 3 & 10 \\ 2 & 1 & 0 & -1 & 2 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 1 & 1 & 2 & 5 \\ 0 & -1 & 0 & -3 & -8 \end{array}\right]$

$\overset{2,3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 & 5 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & -1 & -3 \end{array}\right] \overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 3 & 8 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$\overset{3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & 3 & 8 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ $\therefore$ $x_1 = -3 + 2x_4$

$x_2 = 8 - 3x_4$

$x_3 = -3 + x_4$

$$\therefore \boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3 \\ 8 \\ -3 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 2 \\ -3 \\ 1 \\ 1 \end{bmatrix}}$$

$\boxed{7}$ $x_2 = 2 - 3x_5 - 7x_7$

$x_4 = 8 - 5x_5 - 6x_7$

$x_6 = 9 - 4x_7$

x_1, x_3, x_5, x_7 ARE ALL FREE VARIABLES

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 8 \\ 0 \\ 9 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ -3 \\ 0 \\ -5 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \delta \begin{bmatrix} 0 \\ -7 \\ 0 \\ -6 \\ 0 \\ -4 \\ 1 \end{bmatrix}}$$