MATH 392

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CH 4

[1] (a) $\begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix} = (3)(4) - (-1)(2) = 12 + 2 = \boxed{14}$

(b) $\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = 5 - 6 = \boxed{-1}$

(c) $\begin{vmatrix} \ln 2 & \ln 3 \\ 3 & 2 \end{vmatrix} = 2 \ln 2 - 3 \ln 3 = \boxed{\ln \frac{4}{27}}$

(d) $\begin{vmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{vmatrix} = \cos^2 t + \sin^2 t = \boxed{1}$

(e) $\begin{vmatrix} \sec x & -1 \\ -1 & \sec x \end{vmatrix} = \sec^2 x - 1 = \boxed{\tan^2 x}$

(f) $\begin{vmatrix} -5 & -1 \\ 2 & 3 \end{vmatrix} = -15 + 2 = \boxed{-13}$

[2] (a) $\begin{vmatrix} 3 & 0 & 0 \\ 4 & 5 & 0 \\ -1 & 2 & 6 \end{vmatrix} = 3A_{11} = 3 \begin{vmatrix} 5 & 0 \\ 2 & 6 \end{vmatrix} = 3(30 - 0) = \boxed{90}$

(b) $\boxed{0}$ (ROW OPERATIONS EASILY PRODUCE A ZERO ROW)

(c) $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = 1A_{31} + 2A_{33} = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

$= (4+1) + 2(1-6) = 5 - 10 = \boxed{-5}$

(d) $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 6 \\ 0 & 2 & -2 \end{vmatrix} = 2A_{11} + A_{21} = 2\begin{vmatrix} 1 & 6 \\ 2 & -2 \end{vmatrix} - \begin{vmatrix} -1 & 3 \\ 2 & -2 \end{vmatrix}$

$= 2(-2-12) - (2-6) = \boxed{-24}$

(e) $\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & -1 & 0 & 0 \end{vmatrix} = A_{13} + A_{23} = \underbrace{\begin{vmatrix} 0 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{vmatrix}}_{B} - \underbrace{\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{vmatrix}}_{C}$

$= -B_{13} - (C_{11} + C_{13}) = -\begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} - \left(\begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} \right)$

$= -1 - (-1+1) = \boxed{-1}$

(f) $\begin{vmatrix} -1 & 2 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 \end{vmatrix} \overset{=}{\underset{\sim}{3}} \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 4 & 0 & 1 \\ 0 & 3 & 1 & 2 \end{vmatrix}$

$\overset{=}{\underset{\sim}{3}} \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -4 & 5 \\ 0 & 0 & -2 & 5 \end{vmatrix} \overset{=}{\underset{\sim}{3}} \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -4 & 5 \\ 0 & 0 & 0 & \frac{5}{2} \end{vmatrix} = \boxed{10}$

$\boxed{3}$ (a) $\begin{vmatrix} t & -2 \\ 3 & 12 \end{vmatrix} = 12t + 6 = 0 \implies \boxed{t = -\frac{1}{2}}$

(b) $\begin{vmatrix} 1-t & 0 \\ 3 & -3-t \end{vmatrix} = (1-t)(-3-t) = 0$

$\implies \boxed{t = -3, 1}$

(c) $\begin{vmatrix} 1-t & 1 & 1 \\ 1 & 2-t & 0 \\ 1 & 0 & 2-t \end{vmatrix} = A_{13} + (2-t) A_{33}$

$-(2-t) + (2-t)\left[(1-t)(2-t) - 1\right] = 0$

$$-(2-t) + (2-t)(t^2 - 3t + 1) = 0$$

$$(2-t)\left[-1 + (t^2 - 3t + 1)\right] = 0$$

$$(2-t)(t-3)t = 0 \Rightarrow \boxed{t = 0, 2, 3}$$

(d) $$\begin{vmatrix} t & 0 & 2 \\ 0 & t-1 & 0 \\ 2 & 0 & t \end{vmatrix} = (t-1)\,A_{22} = (t-1)\begin{vmatrix} t & 2 \\ 2 & t \end{vmatrix}$$

$$= (t-1)(t^2-4) = (t-1)(t-2)(t+2) = 0$$

$$\Rightarrow \boxed{t = -2, 1, 2}$$

$\boxed{5}$

$$\text{AREA} = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 3 & 6 & 1 \\ 2 & 2 & 1 \end{vmatrix} = \frac{1}{2}\left| A_{13} + A_{23} + A_{33} \right|$$

$$= \frac{1}{2}\left| \begin{vmatrix} 3 & 6 \\ 2 & 2 \end{vmatrix} - \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 3 & 6 \end{vmatrix} \right|$$

$$= \frac{1}{2}\left| -6 - 4 + 9 \right| = \boxed{\frac{1}{2}}$$

[6] VOLUME $= \langle 1, 0, 2 \rangle \cdot \langle 0, 1, 1 \rangle \times \langle -1, 1, 0 \rangle$

$$= \langle 1, 0, 2 \rangle \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= \langle 1, 0, 2 \rangle \cdot \langle -1, 1, 1 \rangle = -1 + 2 = \boxed{1}$$

[7] $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. $\begin{vmatrix} -a & -b \\ -c & -d \end{vmatrix} = ad - bc$.

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IN GOING FROM A TO $-A$, EVERY ROW IS MULTIPLIED BY -1, CAUSING THE DETERMINANT TO ALSO GET MULTIPLIED BY -1 (SWITCHING SIGNS).

2 ROWS $\longrightarrow$ DET SWITCHES SIGN TWICE (NO CHANGE.)

3 ROWS $\longrightarrow$ DET SWITCHES SIGN THRICE (NEGATIVE.)

[8] (a) $x = \dfrac{\begin{vmatrix} 5 & 2 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}} = \dfrac{-7}{-7} = 1$

$y = \dfrac{\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}} = \dfrac{-14}{-7} = 2$

$\boxed{\begin{array}{l} x = 1 \\ y = 2 \end{array}}$

(b) $$x = \frac{\begin{vmatrix} 2 & 3 \\ -1 & -9 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 4 & -9 \end{vmatrix}} = \frac{-15}{-30} = \frac{1}{2}$$

$$y = \frac{\begin{vmatrix} 2 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 4 & -9 \end{vmatrix}} = \frac{-10}{-30} = \frac{1}{3}$$

$$\boxed{\begin{aligned} x &= \tfrac{1}{2} \\ y &= \tfrac{1}{3} \end{aligned}}$$

(c) $$x = \frac{\begin{vmatrix} 0 & -3 \\ 3 & -9 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & -9 \end{vmatrix}} = \frac{9}{-3} = -3$$

$$y = \frac{\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & -9 \end{vmatrix}} = \frac{3}{-3} = -1$$

$x = -3$

$y = -1$

(d) $$x = \frac{\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{0}{-2} = 0$$

$$y = \frac{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{2}{-2} = -1$$

$$\boxed{\begin{aligned} x &= 0 \\ y &= -1 \end{aligned}}$$

$\boxed{9}$

(a)

$$\begin{vmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix} = A_{43} + A_{44}$$

$$= - \underset{B}{\begin{vmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}} + \underset{C}{\begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & -2 \end{vmatrix}}$$

$$= -(B_{13}) + (-C_{11}) = - \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} - \begin{vmatrix} 1 & -1 \\ -1 & -2 \end{vmatrix}$$

$$= -(-1) - (-3) = \boxed{4}$$

(b) AGAIN

$$\begin{vmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix} \overset{=}{\underset{\sim}{3}} \begin{vmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$$\overset{=}{\underset{\sim}{3}} \begin{vmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} \overset{=}{\underset{\sim}{3}} \begin{vmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & \frac{4}{3} \end{vmatrix} = \boxed{4}$$

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(c) $\begin{vmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 & \boxed{1} \\ 0 & 1 & -1 & \boxed{0} \\ 1 & -1 & -2 & \boxed{0} \\ 1 & 0 & 1 & \boxed{0} \end{vmatrix}$

$= -(1) \begin{vmatrix} 0 & 1 & -1 \\ 1 & -1 & -2 \\ 1 & 0 & 1 \end{vmatrix} = -(1) \begin{vmatrix} \boxed{0 \quad 0 \quad -1} \\ 1 \quad -3 \quad -2 \\ 1 \quad 1 \quad 1 \end{vmatrix}$

ADD THIS COLUMN

TO THIS COLUMN

$= -(1)(-1) \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = -(1)(-1)(4) = \boxed{4}$

(d) $\begin{vmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = A_{14} = -\begin{vmatrix} 0 & 1 & -1 \\ 1 & -1 & -2 \\ 1 & 0 & 1 \end{vmatrix}$

B

ADD THIS COLUMN

TO THIS COLUMN

$$= -\left(B_{31} + B_{33}\right) = -\begin{vmatrix} 1 & -1 \\ -1 & -2 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= -(-3) - (-1) = \boxed{4}$$